Богомолов А.Н., Ушаков А.Н., Богомолова О.А. Определение напряжений в точках контура одиночных подземных горизонтальных горных выработок, находящихся под равномерным давлением, и расчет допустимой глубины их заложения // Construction and Geotechnics. – 2021. – Т. 12, № 3. – С. 117–129. DOI: 10.15593/2224-9826/2021.3.12

Bogomolov A.N., Ushakov A.N., Bogomolova O.A. Determination of stress at the contour points of single underground horizontal mine workings, subject to uniform pressure, and calculation of their permissible laying depth. *Construction and Geotechnics*. 2021. Vol. 12. No. 3. Pp. 117-129. DOI: 10.15593/2224-9826/2021.3.12

CONSTRUCTION AND GEOTECHNICS
Т. 12, № 3, 2021
http://vestnik.pstu.ru/arhit/about/inf/

DOI: 10.15593/2224-9826/2021.3.12
УДК 624.154.1, 624.131.34

ОПРЕДЕЛЕНИЕ НАПРЯЖЕНИЙ В ТОЧКАХ КОНТУРА ОДИНОЧНЫХ ПОДЗЕМНЫХ ГОРИЗОНТАЛЬНЫХ ГОРНЫХ ВЫРАБОТОК, НАХОДЯЩИХСЯ ПОД РАВНОМЕРНЫМ ДАВЛЕНИЕМ, И РАСЧЕТ ДОПУСТИМОЙ ГЛУБИНЫ ИХ ЗАЛОЖЕНИЯ

А.Н. Богомолов, А.Н. Ушаков, О.А. Богомолова

Волгоградский государственный технический университет, Волгоград, Россия

О СТАТЬЕ

Получена: 23 августа 2021
Принята: 10 сентября 2021
Опубликована: 30 сентября 2021

Ключевые слова:

функция комплексного переменного, отображающая функция, горизонтальная горная выработка, напряженное состояние, равномерное давление, коэффициент боковой тяги, глубина заложения горной выработки.

АННОТАЦИЯ

Представлены результаты построения контура подземной горизонтальной горной выработки заданных размеров и геометрической структуры по предложенному авторами методу. Приведено напряженное состояние в точках контуров выработки, сечения которой имеют форму свода с наклонными стенками и эллипса, определены допустимые глубины заложения при заданных значениях равномерного давления, приложенного в точках их контуров. Критерием для определения допустимой глубины заложения является условие отсутствия на контурах выработок точек, в которых нормальные касательные напряжения превышают предел прочности на растяжение и сжатие вмещающей породы.

© **Богомолов Александр Николаевич** – доктор технических наук, профессор, e-mail: banzaritcyn@mail.ru.
Ушаков Андрей Николаевич – кандидат технических наук, доцент, e-mail: miit.vgasu@mail.ru.
Богомолова Оксана Александровна – кандидат технических наук, доцент, ORCID: 0000-0003-1163-6285, e-mail: boazaritcyn@mail.ru.

Alexander N. Bogomolov – Doctor of Technical Sciences, Professor, e-mail: banzaritcyn@mail.ru.
Andrei N. Ushakov – Ph. D. in Technical Sciences, Associate Professo, e-mail: miit.vgasu@mail.ru.
Oksana A. Bogomolova – Ph. D. in Technical Sciences, Associate Professor, e-mail: boazaritcyn@mail.ru.

DETERMINATION OF STRESS AT THE CONTOUR POINTS OF SINGLE UNDERGROUND HORIZONTAL MINE WORKINGS, SUBJECT TO UNIFORM PRESSURE, AND CALCULATION OF THEIR PERMISSIBLE LAYING DEPTH

A.N. Bogomolov, A.N. Ushakov, O.A. Bogomolova

Volgograd State Technical University, Volgograd, Russian Federation

ARTICLE INFO

Received: 23 August 2021
Accepted: 10 September 2021
Published: 30 September 2021

Keywords:
complex variable function, displaying function, horizontal mine working, stress state, uniform pressure, lateral thrust coefficient, laying depth of mine working.

ABSTRACT

The results of constructing the contour of an underground horizontal mine working of given dimensions and geometric structure based on the method proposed by the authors are presented, the stress state at the points of the contours of the working, the cross-sections of which have the shape of a vault with inclined walls and an ellipse, are given, the permissible laying depths are determined at given values of uniform pressure applied at the points of their contours. The criterion for determining the permissible laying depth is the condition that there are no points on the contours of the workings at which the normal tangential stresses exceed the tensile and compressive strength of the enclosing rock.

It is well known that one of the central tasks of geomechanics is the problem of assessing the stress state of rocks around single workings [1–4] in an elastic isotropic massif, moreover, the problem associated with the study of the stress state at the points of the mine working contours is of considerable interest, since the solution of this problem is associated with the problem of their strength.

The geometric structure of the contours of mine workings when solving two-dimensional problems of the linear theory of elasticity [5–8] can be obtained using complex variable functions that carry out a conformal mapping of one of the canonical domains to the domains of interest to us. The mapping functions are polynomials representing the simplest and most well-studied class of functions [9–11].

Let's use the mapping function presented in [12; 13], which looks like this

$$z = \omega(\zeta) = i(A\zeta^{-1} + B\zeta + C\zeta^2 + D\zeta^3), \tag{1}$$

where A, B, C, D – are real numbers; this function maps the interior of the unit circle $|\zeta| < 1$ to the exterior of an infinite simply connected region, the boundary of which is a family of simple closed curves.

The mapping function (1) was previously used by the authors of this article in [14–16] when solving problems related to the study of stresses on the contour of mine workings and to determine the permissible laying depth of horizontal workings with a given cross-sectional configuration.

Consider an underground horizontal working, in which shape of its section is determined using the mapping function (1). Let's assume that this mine working is located at a sufficient depth H, and an internal uniform pressure of intensity p acts along the mine working contour, which makes it possible to consider the working as an underground storage, for example, of hydrocarbons of a given size and configuration.

The purpose of this work is to study the stress state at the points of the contour of an underground horizontal working, built on the basis of the algorithm proposed by the authors, and to determine the permissible laying depth of mine working at various values of the internal uniform tensile pressure acting on this contour.

Let's assume that there are no points on the mine working contour at which the normal tangential stresses exceed the tensile and compressive strengths that is $\sigma_{рас} \leq \sigma_\theta \leq \sigma_{сж}$ [15].

As an example, consider two types of mine workings of a given size: a vault with inclined walls and also a working with an elliptical cross-section.

Let's proceed to the construction of the contour of the working, the cross-section of which is a vault with inclined walls of 4 m height and 5 m width.

Now let's examine a vaulted hole and associate an Oxy coordinate system with it. Let h – be its height, δ – distance from the lower base to the abscissa axis, parameters a and b be the values of half the lengths of the lower and upper bases of the contour (Fig. 1).

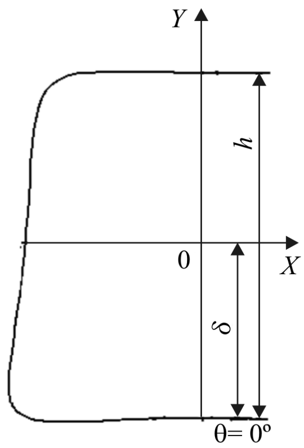


Fig. 1. Problem diagram

In (1) let $\zeta = e^{i\theta}$. Then, separating real component from the imaginary, we obtain the equations for the hole contours in the parametric form:

$$x(\theta) = (A - B)\sin\theta - C\sin 2\theta - D\sin 3\theta,$$
$$y(\theta) = (A + B)\cos\theta + C\cos 2\theta + D\cos 3\theta, \quad (2)$$

where $0 < \theta \leq 2\pi$.

To determine the values of the coefficients A, B, C, D and parameter δ:

1) at $\theta = 0^\circ$ $y = -\delta$, i.e.

$$A + B + C + D = -\delta; \quad (3)$$

2) at $\theta = 180^\circ$ $y = h - \delta$, i.e.

$$-A - B + C - D = h - \delta. \quad (4)$$

Subtracting and adding expressions (3) and (4), we get

$$B = -\left(\frac{h}{2} + A + D\right), \quad (5)$$

$$C = \frac{h}{2} - \delta. \quad (6)$$

3) at $\theta = 90^\circ$ $y = \dfrac{h}{a-b}(a - c + x)$, i.e.

$$C = -\frac{h}{a-b}(a - c + A - B + D). \quad (7)$$

where c – is a real positive number.

Hence, taking into account (6) and (7), we have

$$\delta = \frac{h}{2} + \frac{h}{a-b}\left(a - c + \frac{h}{2} + 2(A + D)\right). \quad (8)$$

Let the points with arguments θ_1, θ_2 be located on the lower base and on the side of the contour respectively.

Then:

$$\text{at } \theta = \theta_1 \quad y = -\delta.$$

Taking into account (2) and (8), we get

$$\alpha_1 A + \beta_1 D = \gamma_1, \tag{9}$$

where

$$\alpha_1 = \frac{2h}{a-b}\left(\cos 2\theta_1 - 1\right), \quad \beta_1 = \cos\theta_1 - \cos 3\theta_1 + \frac{2h}{a-b}\left(\cos 2\theta_1 - 1\right), \tag{10}$$

$$\gamma_1 = \frac{h}{2}\left(1-\cos\theta_1\right) + \frac{h}{a-b}\left(a - c + \frac{h}{2}\right)\left(1 - \cos 2\theta_1\right);$$

at $\theta = \theta_2 \quad y = \dfrac{h}{a-b}(a - c + x).$

Considering (2), we have

$$\alpha_2 A + \beta_2 D = \gamma_2, \tag{11}$$

where

$$\alpha_2 = \frac{2h}{a-b}\left(\cos 2\theta_2 + \sin\theta_2 + \frac{h}{a-b}\sin 2\theta_2\right),$$

$$\beta_2 = \cos\theta_2 - \cos 3\theta_2 + \frac{h}{a-b}\left(\sin\theta_2 - \sin 3\theta_2\right) + \frac{2h}{a-b}\left(\cos 2\theta_2 + \frac{h}{a-b}\sin 2\theta_2\right),$$

$$\gamma_2 = -\frac{h}{2}\left(\cos\theta_2 + \frac{h}{a-b}\sin\theta_2\right) - \frac{h}{a-b}\left(a - c + \frac{h}{2}\right)\left(\cos 2\theta_2 + \frac{h}{a-b}\sin 2\theta_2\right) - \frac{h(a-c)}{a-b}. \tag{12}$$

Solving the system of equations (9) and (11), we find

$$A = \frac{\beta_2\gamma_1 - \beta_1\gamma_2}{\alpha_1\beta_2 - \alpha_2\beta_1}, \tag{13}$$

$$D = \frac{\alpha_1\gamma_2 - \alpha_2\gamma_1}{\alpha_1\beta_2 - \alpha_2\beta_1}, \tag{14}$$

Substituting the obtained values of A and D into the formulas (5), (7), (8), we obtain the values of all coefficients of the mapping function (1) and parameter δ.

The following value is assumed as a width of mine working

$$2|l| = \max_{0\le\theta<2\pi} \left|x(\theta) - x(2\pi - \theta)\right|. \tag{15}$$

Considering the parametric equations of curves (2) describing the contours of underground horizontal workings, it should be noted that the following equalities hold: $x(-\theta) = -x(\theta),\ y(-\theta) = y(\theta)$. Thus, for any values of coefficients A, B, C, D the contours will be symmetric about the ordinate axis, and so let's assume $\theta \in [0, \pi]$. In this case, the width of the working will be equal to twice the value of the function $x(\theta)$ at the point of it's minimum.

Let $\theta = \theta^* -$ be an argument of the function

$$x(\theta) = (A - B)\sin\theta - C\sin 2\theta - D\sin 3\theta, \tag{16}$$

at which this function has a minimum value.

Then argument θ^* can be calculated by the equation

$$12D\cos^3\theta + 4C\cos^2\theta + (B - A - 9D)\cos\theta + 2C = 0. \tag{17}$$

By construction, function (16) takes negative values for any values of the argument $\theta \in (0, \pi)$, moreover, since $x(0) = x(\pi) = 0$, then, according to Rolle theorem [17], we can conclude that function (16) at interval $(0, \pi)$ has a single extremum point, and the equation (17) has a unique real solution.

Note that when calculating numerical values of γ_1 and γ_2, we first assume $c = 0$. Substituting the obtained coefficients of the mapping function (1) or parametric equations of the mine working contour (2) into the equation (17), we obtain the real root $\theta = \theta^*$ of that equation, providing the minimum value of function (16), at that $x(\theta^*) = l$, $l < 0$.

Further, assuming that $\left|a - |l|\right| = c$ and substituting the resulting value c into expressions for γ_1 and γ_2, let's recalculate the values of coefficients A, B, C, D and parameter δ.

Now, using the obtained values of coefficients, let's construct a vault with inclined walls with specific values of its dimensions: $h = 4$ m high and $2l = 5$ m wide.

Let $b = 1$. Then the angle of inclination of the side of mine working contour to its base will be $\alpha \approx 70^\circ$. Let $\theta_1 = 30^\circ$ and $\theta_2 = 120^\circ$. Then, using formulas (13) and (14), considering (10) and (12), we get

$$A = -2{,}438,\ B = 0{,}308,\ C = 0{,}310,\ D = 0{,}130 \tag{18}$$

$$\text{at } \delta = 1{,}690.$$

Substituting coefficients (18) into the equation (17) results in

$$1{,}563\cos^3\theta + 1{,}241\cos^2\theta + 1{,}574\cos\theta - 0{,}620 = 0,$$

and gives a unique solution $\theta = 1{,}268$. Then, according to (2), we have

$$x(\theta) = -2{,}718,\ y(\theta) = -0{,}992.$$

Thus, $|l| = 2{,}718$ and, therefore, $\left|a - |l|\right| = c = 0{,}218$. Substituting the obtained value into the ratios for γ_1 and γ_2, according the formulas (13), (14), (5) and (6) we get

$$A = -2{,}322,\ B = -0{,}203,\ C = 0{,}329,\ D = 0{,}119 \tag{19}$$

$$\text{at } \delta = 1{,}671.$$

Equation (17), taking into account the values of the coefficients (19), takes the form

$$1{,}433\cos^3\theta + 1{,}316\cos^2\theta + 1{,}450\cos\theta - 0{,}658 = 0$$

and has a solution $\theta^* = 1{,}240$. Substituting this value of argument into (2), we get

$$x(\theta^*) = -2{,}524,\ y(\theta^*) = -1{,}047.$$

Now $|l| = 2{,}524$ and, therefore, $\left|a - |l|\right| = 0{,}024$, which clarifies the result.

It is easy to see that it is possible to construct a set of homothetic contours of mine workings, for which it is necessary to multiply the coefficients of the mapping function by the homothety coefficient k<0. The new contours of the workings will have the same configuration, but with linear dimensions changed k times.

Note that mapping function (1) with coefficients (19) performs a conformal mapping of the interior of the unit circle $|\zeta| < 1$ on the obtained area. To verify this fact, it is necessary to establish that the derivative of function (1), regular in the unit circle everywhere, except for the origin, at which it has a simple pole, and for $A \neq 0$ does not vanish at any point on the unit circle $|\zeta| < 1$. That means, that the equation

$$3D\zeta^4 + 2C\zeta^3 + B\zeta^2 - A = 0$$

should not have solutions inside the unit circle $|\zeta| < 1$. Applying Rouche theorem [18], we get the condition

$$|A| > |B| + 2|C| + 3|D|,$$

which holds when the values of the coefficients (19) of the mapping function (1) are substituted into it.

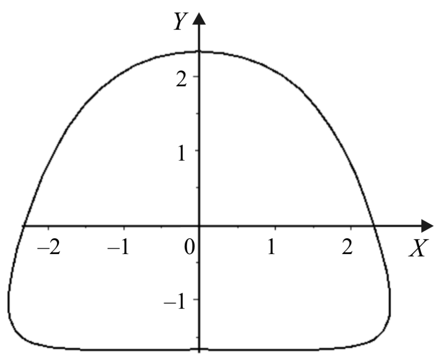


Fig. 2. Cross-sectional shape of mine working (hole)

Thus, the required underground horizontal working with a specified cross-sectional contour is constructed. The mine working contour described by parametric equations (2) with coefficients (19) is shown on Fig. 2.

Following [12], notice that formula, describing the stress state at the points of the mine working contour, the configuration of which is determined using the mapping function (1) under the condition of all-round uniform pressure of a given intensity p applied to the contour, has the form

$$\sigma_\theta = -\frac{\gamma H(F + G\cos\theta + Q\cos 2\theta) - p(K - 4U + (L - 4V)\cos\theta +}{K + L\cos\theta + M\cos 2\theta + N\cos 3\theta + R\cos 4\theta}$$
$$\frac{+(M - 4W)\cos 2\theta - N\cos 3\theta - R\cos 4\theta)}{K + L\cos\theta + M\cos 2\theta + N\cos 3\theta + R\cos 4\theta}, \tag{20}$$

where $$F = (1+\mu)(9D^2 + 4C^2 - A^2) + BS;\ Q = (1+\mu)(A + 3D)B + (3D - A)S; \tag{21}$$

$$G = 2C((1+\mu)(B + 6D) + S);$$

$$S = \frac{(1+\mu)(A + D)B - 2(1-\mu)A^2}{A - D};$$

$$U = \frac{AB^2}{A-D} + 4C^2 + 9D^2; \quad V = \frac{2C}{A-D}[AB + (A-D)(B+6D)]; \tag{22}$$

$$W = \frac{B}{A-D}[6AD - A^2 - 3D^2]; \quad K = A^2 + B^2 + 4C^2 + 9D^2, \quad L = 4C(B+3D),$$

$$M = 2B(3D - A), \quad N = -4AC, \quad R = -6AD \tag{23}$$

where $\gamma -$ volumetric weight of rock; μ – lateral thrust coefficient; H – laying depth of mine working, p – the value of uniform pressure applied to the contour of the hole; moreover, according to [2], we assume that at $p > 0$ the contour of mine working experiences compression of constant value p, and at $p < 0$ – expansion of the same intensity.

Formula (20) was obtained under the condition that the laying depth H of the working is large enough.

Following [4], let's presume

$$H \geq 50R_{\max}, \tag{24}$$

where $R_{\max} -$ the largest linear dimension of the mine working section.

Finding zero values of normal tangential stresses is reduced to solving the equation

$$\begin{aligned} &8Rpt^4 + 4Npt^3 + 2(Q\gamma H + (4W - M - 4R)p)t^2 + (G\gamma H + (4V - L - 3N)p)t + \\ &+(F - Q)\gamma H + (M - 4W + R - K + 4U)p = 0, \end{aligned} \tag{25}$$

where $t = \cos\theta$, $|t| \leq 1$.

Taking into account the results of [16], note that the extreme values of the function $\sigma(\theta)$ can be obtained from the following equations

$$\sin\theta = 0,$$

$$\begin{aligned} &32a_1\cos^5\theta + 16a_2\cos^4\theta + 8(a_3 - 4a_1)\cos^3\theta + 4(a_4 - 3a_2)\cos^2\theta + \\ &+2(3a_1 - 2a_3 + a_5)\cos\theta + (a_2 - a_4 + a_6) = 0, \end{aligned} \tag{26}$$

where

$$\begin{aligned} &a_1 = -RQ\gamma H + (2RM - 4RW)p, \\ &a_2 = -(3/2\,GR + 1/2\,NQ)\gamma H + (3RL - 6RV + MN - 2NW)p, \\ &a_3 = -(GN + 4FR)\gamma H + (8RK + 2NL - 4VN - 16RU)p, \\ &a_4 = (1/2\,QL - 1/2\,MG - 5/2\,GR - 3FN)\gamma H + \\ &+(5RL + 2WL + 6NK - 10RV - 2VM - 12NU)p, \\ &a_5 = (2QK - 2MF - 2GN - 3RQ)\gamma H + \\ &+(6MR + 4LN + 8WK - 12RW - 8VN - 8MU)p, \\ &a_6 = (GK + 3/2\,QL - 5/2\,NQ - 3/2MG - FL)\gamma H + \\ &+(5MN + 6WL + 4VK - 10NW - 6VM - 4LU)p. \end{aligned} \tag{27}$$

Now, using the above relations, let's consider the following problems.

Problem A. Conduct a study of the stress state on the contours of underground horizontal workings with a cross-sections in the shape of a vault with inclined walls and a shape of ellipse under the pressure of uniform all-round expansion of a given intensity at a certain value of the lateral thrust coefficient of the enclosing massif.

Problem B. Determine the permissible values of the laying depth of the underground horizontal workings with a cross-sections in the shape of a vault with inclined walls and the shape of an ellipse under the action of uniform all-round expansion of a given intensity at a given coefficient of lateral expansion of the enclosing massif.

Consider as a granite host rock with volumetric weight $\gamma = 2,5$ t/m³ tinsel strength $R_{\text{рас}} = -1735$ т/m² and compression strength $R_{\text{сж}} = 20400$ т/m². Since the largest linear size of the considered mine working is its width, equal to 5*m,* then, according to (24), we set $H = 250$ m. Then $\gamma H = 625$ т/m².

When solving these problems, let's set the value of the lateral thrust coefficient to $\mu = 0,25$, which corresponds to the value of Poisson's ratio, typically accepted equal to $\nu = 0,20$ [4] for rock.

Solution to problem A

1. Vault with inclined walls

Substituting the values of the coefficients (19) into (1), we obtain an underground horizontal working with a cross-section shown in Fig. 2 and described by parametric equations (2).

Coefficients (22) are independent of the lateral thrust coefficient and are equal to

$$U = 0,601, V = 0,732, W = 0,589;$$
$$K = 5,994, L = 0,738, M = 1,086; N = 3,057, R = 1,663. \quad (28)$$

Substituting the values of the coefficients (19) into formulas (21), we obtain

$$F = -5,320; G = 3,087; Q = 8,994 . \quad (29)$$

As the values of the stresses applied to the mine working contour, we enter:

$$p_1 = 0 \text{ t/m}^2, p_2 = 102 \text{ t/m}^2, \ p_3 = 408 \text{ t/m}^2. \quad (30)$$

Then direct calculations by formula (25) taking into account (28), (29) give us the following values of θ_1 and θ_2 for which $\sigma_\theta = 0$:

$$\text{for } p_1 = 0 \text{ т}/\text{м}^2 \ \ \theta_1 = 0,626, \theta_2 = 2,952;$$

$$\text{for } p_2 = 102 \text{ т}/\text{м}^2 \ \ \theta_1 = 0,655, \theta_2 = 2,847;$$

$$\text{for } p_3 = 408 \text{ т}/\text{м}^2 \ \ \theta_1 = 0,786, \theta_2 = 2,649 .$$

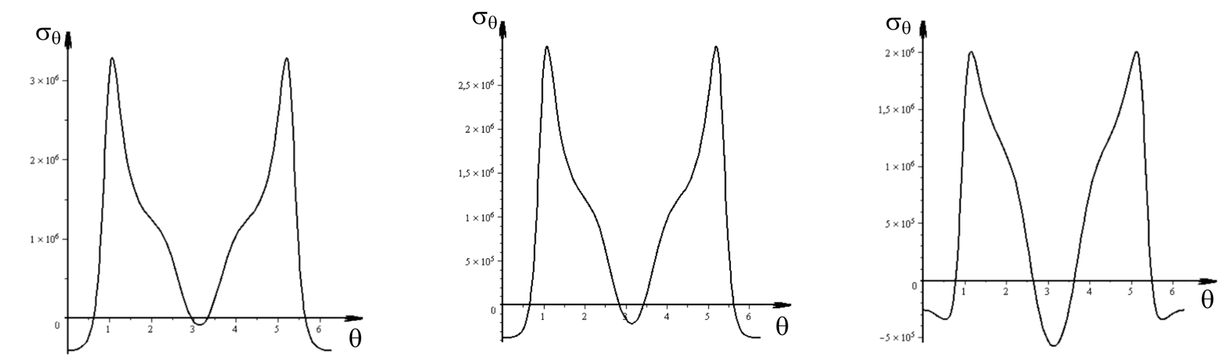
The performed calculations make it possible to identify areas on which tensile stresses act at given values of uniform pressure.

For $p_1 = 0$ t/m² we have $\theta \in (0, 0,626), (2,952, 3,331), (5,657, 2\pi)$;

For $p_2 = 102$ t/m² we have $\theta \in (0, 0,655), (2,847, 3,436), (5,628, 2\pi)$;

For $p_3 = 408$ t/m² we have $\theta \in (0, 0{,}786), (2{,}649, 3{,}634), (5{,}497, 2\pi)$. On other sections of interval $(0, 2\pi)$ compression stresses act.

Diagrams of tangential normal stresses for mine working of considered shape of cross-section at given values of uniform pressure p_1, p_2 and p_3 are shown on Fig. 3.



a *b* *c*

Fig. 3. Diagrams of tangential normal stresses on the contour at:
$p_1 = 0$ t/m² (*a*), $p_2 = 102$ t/m² (*b*), $p_3 = 408$ t/m² (*c*)

2. Mine working with elliptical section

Assuming in the mapping function (1)

$$A = R,\ B = nR,\ C = D = 0, \quad R > 0,\ -1 < n \le 0.$$

we get

$$z = \omega(\zeta) = iR(\zeta^{-1} + n\zeta). \tag{31}$$

Function (31) carries out a conformal mapping of the interior of the unit circle $|\zeta| < 1$ on an infinite plane with an elliptical hole, and the circles $|\zeta| = 1$ corresponds to an ellipse centered at the origin and semiaxes $a = R(1-n)$, $b = R(1+n)$. By setting values of R and n, ellipses of any shape and size can be obtained. If $n = 0$, than ellipse becomes a circle. In the extreme case of $n \to -1$ the ellipse turns into Ox axis segment of $4R$ length between the dots $x = \pm 2R$, and the region turns into an infinite plane with a rectilinear slit.

According to (22)

$$U = n^2R^2, V = 0,\ W = -nR^2, K = (1+n^2)R^2, L = 0, M = -2nR^2, N = R = 0. \tag{32}$$

Taking into account relations (32), formulas (21) give us:

$$F = R^2(1{,}25n^2 - 1{,}5n - 1{,}25), G = 0,\ Q = 1{,}5R^2. \tag{33}$$

Substituting the values of the coefficients (33) into equation (25), we obtain the equation for finding zero values in the form

$$(3\gamma H - 4np)\cos^2\theta + (1{,}25n^2 - 2{,}75 - 1{,}5n)\gamma H + (3n^2 + 2n - 1)p = 0. \tag{34}$$

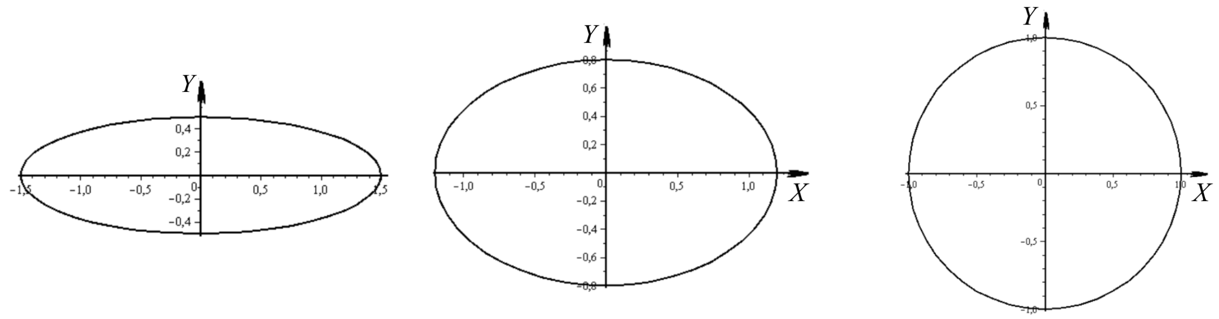
The formula for normal tangential stresses, taking into account the values of the coefficients (32), becomes

$$\sigma_\theta = -\gamma H\left(\frac{1,25n^2-1,5n-1,25+1,5\cos 2\theta}{1+n^2-2n\cos 2\theta}\right)+p\left(1-4\frac{n^2-n\cos 2\theta}{1+n^2-2n\cos 2\theta}\right). \quad (35)$$

Assuming $n=0$ in (35), we obtain the formula for normal tangential stresses for generating a circular cross-section in the form

$$\sigma_\theta = \gamma H(1,25-1,5\cos 2\theta)+p. \quad (36)$$

Let $n_1=-0,5$, $n_2=-0,2$, $n_3=0$. The elliptical holes attained at these values are shown in Fig. 4.



a *b* *c*

Fig. 4. Elliptical shapes of the section at: $n_1=-0,5$ (*a*), $n_2=-0,2$ (*b*), $n_3=0$ (*c*)

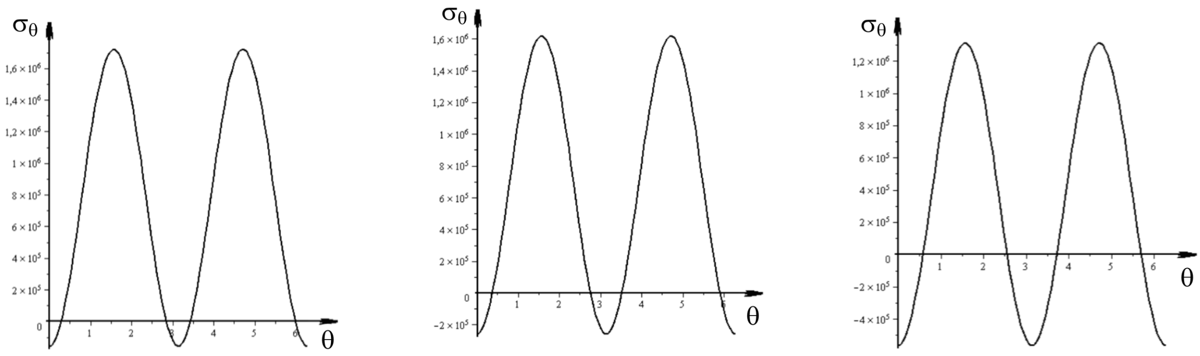
Assuming, like in previous case, $\gamma H=625$ t/m², and, sequentially choosing the values n_1, n_2 and n_3, using equation (34), let's find the areas on which tensile stresses act at given values of uniform pressure (30). We have

for $n_1=-0,5$:
at $p_1=0$ t/m² $\theta\in(0,\ 0,723)\cup(2,419,\ 3,864)\cup(5,560,\ 2\pi)$;
at $p_2=102$ t/m² $\theta\in(0,\ 0,730)\cup(2,411,\ 3,872)\cup(5,553,\ 2\pi)$;
at $p_3=408$ t/m² $\theta\in(0,\ 0,771)\cup(2,371,\ 3,912)\cup(5,512,\ 2\pi)$

for $n_2=-0,2$:
at $p_1=0$ t/m² $\theta\in(0,\ 0,464)\cup(2,678,\ 3,605)\cup(5,819,\ 2\pi)$;
at $p_2=102$ t/m² $\theta\in(0,\ 0,508)\cup(2,634,\ 3,649)\cup(5,775,\ 2\pi)$;
at $p_3=408$ t/m² $\theta\in(0,\ 0,652)\cup(2,489,\ 3,794)\cup(5,631,\ 2\pi)$

for $n_3=0$:
at $p_1=0$ t/m² $\theta\in(0,\ 0,293)\cup(2,849,\ 3,434)\cup(5,990,\ 2\pi)$;
at $p_2=102$ t/m² $\theta\in(0,\ 0,380)\cup(2,761,\ 3,522)\cup(5,903,\ 2\pi)$;
at $p_3=408$ t/m² $\theta\in(0,\ 0,581)\cup(2,561,\ 3,722)\cup(5,702,\ 2\pi)$;
compressive stresses act on other segments of the interval $(0,\ 2\pi)$.

As an example, below are diagrams of normal tangential stresses for wine working with circular cross-section at given values of uniform pressure (Fig. 5).



a *b* *c*

Fig. 5. Diagrams of normal tangential stresses for mine working with circular cross-section at: $p_1 = 0$ t/m² (*a*), $p_2 = 102$ t/m² (*b*), $p_3 = 408$ t/m² (*c*)

Solution to problem B

1. Vault with inclined walls

Let $p_1 = 0$. Then, substituting (28) and (29) into (27) and (26), we obtain an equation for the extreme values of the function $\sigma(\theta)$, the root of which is the value $\theta_1 = 1{,}069$.

By attaching values $\theta_2 = 0$ and $\theta_3 = \pi$ to it, we get

$$\sigma_{\min}(\theta) = \sigma_\theta(\theta_2) = -0{,}539\gamma H, \ \sigma_{\max}(\theta) = \sigma_\theta(\theta_1) = 4{,}307\gamma H .$$

Based on the results of [14; 15], we obtain

$$H_{\text{рас}} \approx 1287 \text{ m}, \ H_{\text{сж}} \approx 1894 \text{ m}.$$

Therefore, at the depth $H > \min(H_{\text{рас}}, H_{\text{сж}}) \approx 1287$ m the considered horizontal working in a rock massif, folded by granite, loses strength.

Linear dimensions of the working, according to (24), should not exceed $2l = 25{,}74$ m, $h = 20{,}6$ m.

Let a uniform confining the sile pressure of the $p_2 = 102$ t/m² magnitude act at the points of the mine working contour.

Consider the expression

$$-\frac{\gamma H(F + G\cos\theta + Q\cos 2\theta) - p(K - 4U + (L - 4V)\cos\theta +}{K + L\cos\theta + M\cos 2\theta + N\cos 3\theta + R\cos 4\theta}$$
$$\frac{+(M - 4W)\cos 2\theta - N\cos 3\theta - R\cos 4\theta)}{K + L\cos\theta + M\cos 2\theta + N\cos 3\theta + R\cos 4\theta} = R_{\text{рас}}, \tag{37}$$

where $R_{\text{рас}}$ – tensile strength of the rock as a function of the mine working laying depth $H(\theta)$.

By differentiating expression (37), and by obtaining for determining the extreme values of the function $H(\theta)$ together with equation $\sin\theta = 0$ trigonometric equation of the ninth degree, let's look for the minimum value of the function $H(\theta)$ near the value of the argument at which the depth function reaches a minimum at $p_1 = 0$. This value at $p_2 = 102$ t/m² is $\theta = 0$. And so, the permissible laying depth of the vault with inclined walls at $p_2 = 102$ t/m² equals $H \approx 1314$ m.

Now let the internal uniform tensile pressure at the points of the working contour be $p_3 = 408$ t/m². The result of caring out the necessary calculations, is that the permissible laying depth of the considered horizontal working is equal to $H \approx 1397$ m.

2. Mine working with elliptical section

Let $p_1 = 0$. Taking into account relations (27), (32), and (33) on the interval $0 \le \theta \le \pi$ equation (26) results in value $\theta_1 = \pi/2$. With $\theta_2 = 0$ and $\theta_3 = \pi$ we obtain values of arguments for which function $\sigma = \sigma(\theta)$ has extreme values.

Further, let $n_1 = -0,5$. Then

$$\sigma_{\min}(\theta) = \sigma_\theta(\theta_2) = \sigma_{\min}(\theta_3) = -0,583\gamma H, \quad \sigma_{\max}(\theta) = \sigma_\theta(\theta_1) = 6,750\gamma H .$$

Taking into account the results of [14; 15], we obtain

$$H_{\text{рас}} \approx 1189 \text{ m}, \quad H_{\text{сж}} \approx 1208 \text{ m} .$$

Therefore, at the laying dept $H > 1189$ m the considered mine working in the rock massif, folded by granite, loses its strength. According to (24), the linear dimensions of the working should not exceed $a = 1,5R$ m, $b = 0,5R$ m, where $R = 15,85$ m.

Carrying out further calculations similar to those done for the previous case, we obtain:

for $n_1 = -0,5$:

at $p_2 = 102$ т/$м^2$ $H = 1212$ m;

at $p_3 = 408$ т/$м^2$ $H = 1282$ m;

for $n_2 = -0,2$:

at $p_1 = 0$ т/$м^2$ $H = 1665$ m;

at $p_2 = 102$ т/$м^2$ $H = 1632$ m;

at $p_3 = 408$ т/$м^2$ $H = 1534$ m;

for $n_3 = 0$:

at $p_1 = 0$ т/$м^2$ $H = 2775$ m;

at $p_2 = 102$ т/$м^2$ $H = 2612$ m;

at $p_3 = 408$ т/$м^2$ $H = 2122$ m.

All problems have been solved.

References

1. Turchaninov I.A., Iofis M.A., Kasparyan E.V. Basics of Rock Mechanics. Leningrad, Nedra, 1989, 488 p.

2. Bulychev N.S. Mechanics of underground structures. Moscow, Nedra, 1989, 270 p.

3. Glushko V.T., Gavelya S.P. Assesment of the tensly-deformed rock massifs. Moscow, Nedra, 1986, 221 p.

4. Tsimbarevich P.M., Rock mechanics. Moscow, Ugletechizdat, 1948, 184 p.

5. Muskhelishvili N.I. Some basic tasks of mathematical theory of elasticity. Moscow, Science, 1966, 708 p.

6. Kolandia A.I. Mathematical methods of two-dimensional elasticity. Moscow, Science, 1973, 304 p.

7. Khan H. Theory of elasticity. The basics of the linear theory of its application. Moscow, Peace, 1988, 344 p.

8. Paron V.Z., Perlin P.I. Methods of mathematical theory of elasticity. Moscow, Science, 1981, 688 p.

9. Filchakov P.F. Approximate conformal mapping methods. Kiev, Naukova Dumka, 1964, 536 p.

10. Filchakova V.P. Conformal mappings of regions of a special type. Kiev, Naukova Dunka, 1972, 252 p.

11. Savin G.N. Distribution of stresses near the holes. Kiev, Naukova Dumka, 1968, 888 p.

12. Tsvetkov V.K. Determination of the shapes of sections of mine workings with specified stresses on the contours. *Physical and technical problems of mining*, 1986, no. 2, pp. 24–29.

13. Tsvetkov V.K. Calculation of rational parameters of mine workings. Moscow, Nedra, 1993, 251 p.

14. Bogomolov A.N., Ushakov A.N. Methods of the theory of functions of a complex variable in problems of geomechanics. Volgograd, VSACU, VSSPU "Change," 2014, 227 p.

15. Bogomolov A.N., Bogomolova O.A., Ushakov A.N. Determination of the laying depth of the horizontal working based on the analysis of the stress state of the enclosing massif. *Bases, foundation and soil mechanics*, 2017, no. 5, pp. 2 – 9.

16. Bogomolov A.N., Bogomolova O.A., Ushakov A.N. On stresses at the contour points of single underground workings of various cross-sections, subject to all-round uniform pressure. *Vestnik PNIPU. Stroitel'stvo and arkhitektura*, 2018, vol. 9, iss. 3, pp. 54–70. DOI: 10.15593/2224-9826/2018.3.06.

17. Nikolski S.M. Mathematical analysis course. Moscow, Science, 1990, 528 p.

18. Sidorov U.V., Fedoruk M.V., Schabunin M.I. Lectures on the theory of functions of a complex variable. Moscow, Science, 1989, 480 p.