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REVIEW OF ANALYTICAL AND EMPIRICAL METHODS FOR CALCULATING SOIL SURFACE SETTLEMENT DURING SHIELD TUNNELING

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ABSTRACT

Tunneling by the shield method often causes deformations in the soil mass and on the surface. With the active development of tunnel boring in the twentieth century, deformations of soil surface and structures have become very relevant topics for the safety of closed work. From the foundations of rock mechanics firstly appeared analytical methods with the consideration of the theories of elasticity for space and half-space. And after that, with reference to the accumulated experience and analysis of field monitoring results a complex of empirical methods appeared. Both groups of methods are still of applied importance and, at times, continue to be improved. The aim of the work is a review and comparative analysis of analytical and empirical methods of various authors for determining settlement and distance to the inflection point of the settlement curve (i_x) , as well as their systematization with reference to their appearance and the features of the development of this field of geotechnics.

With the help of a comprehensive comparative and content analysis of various approaches to determining the settlement of soil surface and the distance to the inflection point of the settlement curve (ix) in tunnel boring, the paper presents the main classification of these analytical and empirical methods in their chronological appearance.

The analysis of some examples of comparison of analytical and empirical methods for calculating the values of soil surface settlement in the sources of different years provides coverage of the absolute majority of studies from the moment of their appearance for taking into account tunnel-boring operations. Comparative results of available research are presented and some approaches of the most cited studies are analyzed.

An extensive review of analytical and empirical methods of the soil surface and structures settlement during tunnel-boring shows that these methods can no longer always meet all the conditions, approaches and standards of modern design methods for geotechnical and tunnel construction tasks. However, understanding the vector of development of these groups of methods provides insight into the development of engineering thought and the accumulation of statistical material of field measurements of half a century tunneling experience.

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ОБЗОР АНАЛИТИЧЕСКИХ И ЭМПИРИЧЕСКИХ МЕТОДОВ РАСЧЕТА ОСАДОК ПОВЕРХНОСТИ ГРУНТА ПРИ ЩИТОВОЙ ПРОХОДКЕ

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тоннели, тоннелепроходческие работы, щитовая проходка, аналитические методы осадок, эмпирические методы осадок, кривая перегиба осадок, зона влияния, осадка земной поверхности, осадка сооружения, потеря грунта, мониторинг, дисперсный грунт. Тоннельные проходки щитовым методом зачастую вызывают деформации в массиве грунта и на поверхности. С активным развитием тоннелепроходческих работ в XX в. деформации земной поверхности и сооружений стали очень актуальными темами для безопасности проведения закрытых работ. Из основ горной механики с учетом теорий упругости для пространства и полупространства сначала появились аналитические методы. А уже с учетом накопленного опыта и анализа полевых результатов мониторинга появилась группа эмпирических методов. Обе группы методов и по сей день имеют прикладное значение и порой продолжают совершенствоваться. Целью работы является обзорный и сравнительный анализ аналитических и эмпирических методов различных авторов определения осадок и расстояния до точки перегиба кривой осадок (*i*_x), а также их систематизация с учетом их появления и особенностей развития данной области геотехники.

С помощью комплексного сравнительного метода и контент-анализа различных подходов к определению осадок земной поверхности и расстояния до точки перегиба кривой осадок (*i*_x) при тоннелепроходческих работах представлена основная классификация данных аналитических и эмпирических методов в хронологическом порядке их появления.

Проведенный анализ некоторых примеров сравнения аналитических и эмпирических методов расчета значений осадок земной поверхности источников различных лет обеспечивает охват абсолютного большинства исследований от момента их появления для учета тоннелепроходческих работ. Приводятся сравнительные результаты доступных исследований и анализируются некоторые подходы наиболее цитируемых исследований.

Обширный обзорный анализ аналитических и эмпирических методов осадок поверхности земли и сооружений при тоннелепроходческих работах показывает, что данные методы уже не всегда могут отвечать всем условиям, подходам и стандартам современных методик проектирования для задач геотехники и тоннелестроения. Однако понимание вектора развития этих групп методов дает хорошее представление о ходе развития инженерной мысли и накоплении статистического материала полевых измерений полувекового опыта тоннелепроходческих работ.

Introduction

Even before the era of industrialization in Europe old collectors in many cities including Paris (France), Vienna (Austria), London (UK), Amsterdam (Netherlands) and Berlin (Germany) were built by hand with mining methods. These underground tunnel systems contain the cities' utilities, such as water, sewer, gas, electrical and telephone cables, and compressed air pipes. The history of the Parisian collectors, for example, generally dates back to antiquity, but due to pollution they fell into disrepair during the Middle Ages. However, the total length of Parisian collectors even now is about 2500 km and, as can be seen from history, they were built for more than a dozen centuries. And accounting the rapid growth of industrialization of large cities, the development of transport and engineering infrastructure required growth even at a faster pace. Due to the need for fast, high-quality and safe work in a closed method, the first tunnel boring machine was invented by the English engineer Mark Brunel in 1825. It was used in the construction of a tunnel under the River Thames. And since it was used for tunneling under the river everything what happened above the tunnel was of little importance. However, later it turned out that the deformations from tunneling by

the shield method and, as a result, the potential damage from them to underground utilities, structures and overlying buildings in the zone of influence can be significant and, even catastrophic.

One of the first works in the field of closed tunneling, which is still relevant today, was the publication of the American-Canadian Prof. R.B. Peck [1] in the part of the report devoted to tunneling in dispersed soils. And nevertheless this work is empirical, i.e. it has some regression dependence on the basis of the obtained experience and field data of settlement measurements. Analytical methods utilizing elasticity theory as applied to geomechanics appeared somewhat earlier than this work. It may be noted that in more than 50 years of experience in the development of analytical and empirical methods, many variants as well as different variations of the early methods have been proposed by different authors. But in the context of further technological progress and the appearance of powerful computers another group of methods, the most prevailing at the moment, namely numerical methods, has been actively developing since the early 80s of the 20th century. It is numerical methods for changing the stress-strain state (SSS), using the programs of finite element method (FEM) and finite difference (FDM) analysis that are used for the majority of modern studies of tunnel boring works, and all authors rely on one or another analytical and empirical methods of various researchers to confirm their results. So, it is especially important to consolidate and make actual the available data and many years of experience in determining the values of surface settlement and the size of the possible zone of influence which determines the amount of deformations of structures and the quality of these deformations by analytical and empirical methods since they are the main and comparative basis for current research in this field of geotechnics.

The purpose of this paper is to make a historical review and comparative analysis of analytical and empirical methods in predicting the settlement of surfaces, buildings and structures, as well as determining the size of the possible zone of influence.

Analytical methods

Such authors as Jeffery G.B. [2]; Mindlin, R.D. [3]; Limanova Y.A. [4]; Genieva G.A. [5]; Sagaseta S. [6]; Verruijt A. & Booker J.R. [7, 8]; Verruijt A. [9]; Loganathan N. & Poulos H.G. [10]; Gonzalez C. & Sagaseta S. [11]; Yarovoy Yu.I. [12]; Wang J.G. et al. [13]; Heina A.M. [14]; Elgaeva V.S. [15] and others were engaged in the study of analytical methods at different times. Various soil models, in particular elastic models, are used to determine the settlement of the surface and the effect on the foundations of structures during the construction of deep or shallow tunnels, and sometimes it is permissible to use solutions for infinite media, if the stress distribution in the foundation base can be neglected.

One of the earliest known works of analytical methods for predicting the parameters of soil shift trough in urban areas is the method of Professor Y.A. Limanov [4], developed for the construction of a deep underground railway in St. Petersburg and based on experimental field studies and the theory of elasticity. Yu.A. Limanov solved the problem of changing the SSS during circular development of rock mass of the Cambrian clays, represented as a linearly deformable isotropic half-plane with a distributed load at the upper boundary by the reduced weight value of weak rocks of Quaternary deposits. For simplification, the heavy isotropic half-plane is represented by an unweighted half-plane with a circular hole, and "removable stresses" of the value P with the opposite sign are applied to its contour. The solution of the problem is determined with the use of the bipolar transformations coordinates of G.B. Jeffery [2] for the displacements of an eccentric cylinder with outer circle curvature tending to "0". Taking into account the correspond-

ing initial and boundary conditions (Fig. 1, *a*), Y.A. Limanov [4] obtained expressions for vertical and horizontal displacements of the hole contour and the outer border of the half-plane:

$$\eta(x) = \eta_0 \left(1 - \frac{x}{L} \right)^4 e^{4\frac{x}{L}}; \tag{1}$$

$$\eta_0 = \frac{F}{L}, \quad \text{где} \quad L = 2a + h_1 \text{tg} \left(45 - \frac{\varphi}{2} \right); \tag{2}$$

$$a = \sqrt{h_2^2 - r^2}; (3)$$

where x – abscissa of the soil surface; F – area of half-trough settlement of the contact line of Cambrian clays and weak rocks; h_1 – thickness of weak rocks; h_2 – distance from the rock contact line to the center of the tunnel; 2a – length of the half-trough settlement of the rock contact line; φ – angle of internal friction of soil; r – radius of the workings.

The area *F* is determined by a formula that can be reduced to the form:

$$F = 8\pi \left(1 - \mu^2\right) \frac{\sigma h_2 r^2}{E_0 2a};$$
(4)

where μ – Poisson coefficient of Cambrian clays; σ – Average value of natural stresses acting along the contour of the workings; E_0 – modulus of deformation of Cambrian clays.

Another analytical work was the work of G.A. Geniev [5] (Fig. 1, *b*) for non-cohesive dispersed soils. Settlement of the surface of the soil mass in closed workings is determined as:

$$v_0 = H - \left\{ H - 4R^2 \left(\cos^2 \varphi - \cos^2 \alpha \right)^{0.5} \frac{\sin^3 \alpha}{\cos^2 \varphi} \right\}^{0.5};$$
(5)

where *H* – depth of the subsidence trough sector; *R* – radius of the workings; $\varphi = \frac{\pi}{4} + \frac{\rho}{2}$, where ρ is the angle of internal friction of the soil.



Fig. 1. Calculation scheme for assessing surface settlements: *a* – by the method of Y.A. Limanov [4] with a settlement curve by analogy with S.G. Avershin [16]; *b* – by the method of G.A. Geniev [5] Рис. 1. Расчетная схема для оценки осадок поверхности: *a* – методом Ю.А. Лиманова [4] с кривой осадок по аналогии с С.Г. Авершиным [16]; *b* – методом Г.А. Гениева [5]

In the study of Elgaev V.S. [15], based on the theory of reciprocity, the following method for assessing deformations of the soil surface was proposed:

$$u_{z} = \frac{2\Delta \cdot z_{0}R}{r^{2}} \left(\frac{1+v^{2}}{1-v^{2}}\right);$$
(6)

where u_z – vertical displacements of soil surface; Δ – gap between tunnel lining and workings; z_0 – depth of the tunnel axis; R – radius of the workings; r – distance from the tunnel axis to the point of displacements determination; v – coefficient of transverse deformation of the soil (Poisson's ratio).

However, the solutions of the plane problem of elasticity for space and half-space are more widely used since in shallow tunnels the free surface strongly influences the distribution of stresses and, consequently, displacements. Applying the infinite domain solution and the theory of functions of complex variables of Verruijt A. & Booker J.R. [7] an analytical solution for the half-plane was obtained: Normal stresses and shear stresses are applied around the circumference of the cavity, so that the surface tensions along this boundary are nullified. The problem is solved by superimposing 3 partial solutions and, at that, for the second problem it was found a standard solution by Melan E. [17]. These solutions are agreed upon the superposition of three solutions with the use of complex variable method in the following way:

$$u_{x} = -\varepsilon a^{2}x \left(\frac{1}{r_{1}^{2}} + \frac{1}{r_{2}^{2}}\right) + \delta a^{2}x \left\{\frac{\left(x^{2} - kz_{1}^{2}\right)}{r_{1}^{4}} + \frac{\left(x^{2} - kz_{2}^{2}\right)}{r_{2}^{4}}\right\} - \frac{2\varepsilon a^{2}x}{m} \left(\frac{1}{r_{2}^{2}} + \frac{2mzz_{2}}{r_{2}^{4}}\right) - \frac{4\delta^{2}xH}{m+1} \left\{\frac{z_{2}}{r_{2}^{4}} + \frac{mz\left(x^{2} - 3z_{2}^{2}\right)}{r_{2}^{6}}\right\};$$

$$u_{z} = -\varepsilon a^{2} \left(\frac{z_{1}}{r_{1}^{2}} - \frac{z_{2}}{r_{2}^{2}}\right) + \delta a^{2} \left\{\frac{z_{1}\left(kx^{2} - z_{1}^{2}\right)}{r_{1}^{4}} + \frac{z_{2}\left(kx^{2} - z_{2}^{2}\right)}{r_{2}^{4}}\right\} - \frac{1}{2}\left(\frac{z_{1}}{r_{2}^{4}} - \frac{z_{2}}{r_{2}^{4}}\right) + \frac{1}{2}\left(\frac{z_{1}}{r_{2}^{4}} - \frac{z_{2}}{r_$$

 $-\frac{2\varepsilon a^{2}}{m}\left(\frac{(m+1)z_{2}}{r_{2}^{2}}+\frac{mz(x^{2}-z_{2}^{2})}{r_{2}^{4}}\right)-2\delta a^{2}H\left\{\frac{x^{2}-z_{2}^{2}}{r_{2}^{4}}+\frac{m}{m+1}\frac{2zz_{2}\left(3x^{2}-z_{2}^{2}\right)}{r_{2}^{6}}\right\};$ (8)

where δ – infinite soil deformation caused by ovalization of the tunnel lining; $z_1 = z - H$; $z_2 = z + H$; $r_1^2 = x^2 + z_1^2$; $r_2^2 = x^2 + z_2^2$; m = 1/(1-2v); k = v/(1-v); v – the transverse deformation coefficient of the soil (Poisson's ratio).

A little later, the same Verruijt A. & Booker J.R. [8] in their work on the "buoyancy" of tunnels in soft (structurally unstable) soils presented a more optimised solution:

$$u_{x} = \frac{P}{8\pi\mu(1-\nu)} \left\{ \frac{xy_{1}}{r_{1}^{2}} - \frac{xy_{2}}{r_{2}^{2}} \right\} - \frac{P(1-2\nu)}{2\pi\mu} \arctan\left(\frac{x}{y_{1}}\right); \qquad (9)$$

$$+ \frac{Px(y+h)}{2\pi\mu r_{1}^{2}} - \frac{Phx(r_{1}^{2}+2yy_{1})}{4\pi\mu(1-\nu)r_{1}^{4}}$$

$$u_{z} = \frac{P}{8\pi\mu(1-\nu)} \left\{ (3-4\nu)\log\left(\frac{r_{1}}{r_{2}}\right) + \frac{x^{2}}{r_{1}^{2}} - \frac{x^{2}}{r_{2}^{2}} \right\} - \frac{P(1-\nu)}{\pi\mu} \log(r_{1})$$

$$+ \frac{Py_{1}^{2}}{2\pi\mu r_{1}^{2}} - \frac{Phy(x_{1}^{2}-y_{1}^{2})}{4\pi\mu(1-\nu)r_{1}^{4}} + u_{0}$$
(10)

where $P = \gamma \pi r^2$ – the weight of the excavated soil from the tunnel, acting in the upward direction, and applied at the point x = 0; y = -h inside the half-plane y < 0; $y_1 = y - h$; $y_2 = y + h$; $r_1 = \sqrt{x^2 + y_1^2}$; $r_2 = \sqrt{x^2 + y_2^2}$; μ – shear elastic modulus; v – coefficient of transverse soil deformation (Poisson's ratio); u_0 – arbitrary constant associated with arbitrary displacement of a solid.

In the case of uniform perimeter soil loss, Sagaseta C. [6] presented the following solution for calculating surface deformations:

$$\begin{cases} S_{x_{0}} = -\frac{V_{s}}{2\pi} \frac{x}{x^{2} + h^{2}} \left[1 + \frac{y}{\left(x^{2} + y^{2} + h^{2}\right)^{0.5}} \right] \\ S_{y_{0}} = \frac{V_{s}}{2\pi} \frac{1}{\left(x^{2} + y^{2} + h^{2}\right)^{0.5}} \\ S_{z_{0}} = \frac{V_{s}}{2\pi} \frac{h}{x^{2} + h^{2}} \left[1 + \frac{y}{\left(x^{2} + y^{2} + h^{2}\right)^{0.5}} \right] \end{cases}$$
(11)

Loganathan N. & Poulos H.G. [10] suggested that for water-saturated soils the coefficient of transverse ground deformation (Poisson's ratio) should be used equal to $v_u = 0.5$, and lateral pressure coefficient Jâky [18] $K_0 = 1.0$ when determining settlements over a short time interval, where long-term losses of the circular shape of the tunnel



Fig. 2. Surface deformations during tunnelling by C. Sagaseta [6] Рис. 2. Деформации поверхности при тоннелировании по C. Sagaseta [6]

lining can be neglected and the ground deformations can be taken as $\delta = 0$. Simplified expressions for determining the soil deformation considering only uniformly radial soil loss are proposed as follows:

$$u_{x} = -\varepsilon a^{2} x \left(\frac{1}{x^{2} + (z - H)^{2}} + \frac{1}{x^{2} + (z + H)^{2}} - \frac{4z(z + H)}{\left[x^{2} + (z + H)^{2}\right]^{2}} \right).$$
(12)

$$u_{z} = -\varepsilon a^{2} \left(-\frac{z-H}{x^{2} + (z-H)^{2}} + \frac{z+H}{x^{2} + (z+H)^{2}} - \frac{2z \left[x^{2} - (z+H)^{2}\right]}{\left[x^{2} + (z+H)^{2}\right]^{2}} \right).$$
(13)

As mentioned above, Sagaseta S. [6]; Verruijt A. & Booker J.R. [7]; Loganathan N. & Poulos H.G. [10]; Gonzalez C. & Sagaseta C. [11] use radial "soil loss" with constant radial displacement around the tunnel to determine the magnitude of "soil loss". To determine the "soil loss" it is used other techniques and methods as well, but the best known methods and definitions are as follows:

1. Radial shrinkage (uniform radial soil loss) according to Verruijt A. & Booker J.R. [7]:

$$\varepsilon_1 = \frac{u_0}{a}; \tag{14}$$

2. Specific soil loss according to Gonzalez C. & Sagaseta C. [11]:

$$\varepsilon_2 = \frac{2\pi a u_0}{\pi a^2}; \tag{15}$$

3. Equivalent value of "soil loss» according to Loganathan N. & Poulos H.G. [10]:

$$\varepsilon_{3} = \frac{\pi \left(a - \frac{g}{2}\right)^{2} - \pi a^{2}}{\pi a^{2}} = \frac{4ga - g^{2}}{4a^{2}}.$$
 (16)

where g – gap parameter used in numerical methods (gap method).

4. Modified equivalent value of "soil loss" according to Loganathan N. & Poulos H.G. (1998) [10]:

$$\varepsilon_4 = \varepsilon_3 \cdot \exp\left\{-\left[\frac{1.38x^2}{\left(H+a\right)^2} + \frac{0.69z^2}{H^2}\right]\right\}.$$
(17)

One of the recent studies on this problem was the work of Park K.-H. [19], where the author considered 4 variants of boundary conditions (B.C.) and, respectively, deformation forms.



Fig. 3. Boundary conditions (B.C.) of a specified displacement by K.-H. Park [19] Рис. 3. Граничные условия (Г.У.) заданного перемещения по K.-H. Park [19]

It is mentioned in the study that the variant of B.C.-2 deformation shape shown in Fig. 3, d, sufficiently correctly describes the soil surface settlement according to the field data and in comparison with the known computational models of Sagaseta S. [6] and Verruijt A. [6]. [6] and Verruijt A. & Booker J.R. [7] – B.C.-1; Loganathan N. & Poulos H.G. [10] – B.C.-4.

In the work of Wang J.G. et al. [13], the superposition method is used to calculate the value of the settlement surface for two parallel tunnels in the form of the expression:

$$S_{t} = S_{\max A} e^{-\frac{(x+L/2)^{2}}{2l_{A}^{2}}} + S_{\max B} e^{-\frac{(x-L/2)^{2}}{2l_{B}^{2}}} - S_{AB}.$$
 (18)

where $S_{AB} = 0$ – without regard to interaction; L – the distance between two tunnels. The larger the distance between the tunnels the smaller the settlement of the soil mass between them, and if the tunnels have the same diameters and "soil losses", then $S_{max A} = S_{max B} \bowtie l_A = l_B$.

In the study by Hein A.M. [14] it was developed an analytical method for estimating the soil surface settlement based on the reciprocity theorem formulated as: if a force F applied in the α direction at some point A of an elastic, anisotropic, inhomogeneous space causes a displacement equal to u at another point B in the β direction, then the same force F applied at point B in the β direction will cause a displacement equal to u at point A in the α direction. The method is based on the known analytical solution of the displacement of points of an elastic half-space of soil under the influence of vertical forces. Calculations of vertical displacements of the soil surface are

derived from the function of displacement of the cavity contour due to the average radial displacements of the tunnel lining from the force *F* applied inside the closed workings:



Fig. 4. Calculation scheme for estimation of settlement using the reciprocity theorem according to the work of A.M. Hein [14] Рис. 4. Расчетная схема для оценки осадок с помощью теоремы взаимности по работе A.M. Хейна [14]



Empirical methods



As mentioned above, R.B. Peck [1] was the first to propose a closely approximating curve of vertical soil deformations in the transverse direction for assessing the effect of tunnels for closed tunnel penetrations. On the basis of field observations the curve of vertical soil deformations for

tunnelling, described by the Gaussian function, and the predictive trough of the impact of tunneling, R.B. Peck expressed as a dependence:

$$S_{\nu}(x) = S_{\nu,\max} \cdot e^{-\frac{x^2}{2t_x^2}}.$$
 (20)

where $S_v(x)$ – displacement of the soil surface depending on the distance to the tunnel axis; $S_{v,max}$ – maximum settlement of soil surface above the longitudinal axis of the tunnel, mm; x – distance from the center of the tunnel to the surface along the vertical through the axis of the tunnel, m; i_x – distance from the center of the tunnel horizontally to the bending point of the settlement curve, m.

Distance to the bending point (i_x) on the graph the surface settlement above the tunnel is determined by the following formula:

$$\frac{i_x}{R} = \left(\frac{z_0}{2R}\right)^n$$
: where $n = 0, 8 - 1, 0,$ (21)

where n = 0,8-1,0; z_0 – depth of the tunnel axis (closed working), m; R – radius of tunnel (closed working), m.

It should be noted that studies of closed workings were conducted long before the work of Peck R.B. [1]. [1], however, either their scale was not massive or public or they did not have a large base of field studies. For example, in the works of Knothe S. [20] and Cording E.J. & Hansmire W.H. [21] formulas for determining half the width of the influence trough for closed workings by manual or semi-mechanized method, applied to the mining method, was presented.

Further, the search for the definition of i_x and its proportion in the definition of a trough has become more active and landmark works were noted by Atkinson J.H. & Potts D.M. & Potts D.M. [22]; Glossop N.H. [23] in co-authorship with Mair R.J. et al. [24]. Clough G.W. & Schmidt B. [25] were the first to present data indicating that i_x is also tunnel diameter dependent, especially for small depth-to-diameter (C/D) ratios and more dependent on soil type; a study by O'Reilly M.R. & New B.N. [26] defined the direction for decades to come and consolidated the importance of the V_L (relative volume "loss") parameter. Great research was carried out by Attewell P.B. et al. [27]; Herzog M. [28]; Leach G. [29]; Rankin W.J. [30]; Uriel A.O. & Sagaseta C. [31]; Arioglu E. [32]; Mair R.J. et al. [33]; Lee C.J. et al. [34]. Based on analyses of past experience, Möller S.C. [35] concluded that expansion and swelling due to unloading can lead to soil expansion, so that $V_s < V_{sl}$. However, the difference is still small and this simplification makes it possible to adopt the material balance equation (22). The ground loss enclosed between the day surface and the curve on the Gauss function depends more or less linearly on the tunnel volume, and Möller S.C. introduces the concept of ground loss ratio (*GLR*).

$$V_{s} \approx V_{sl} \,. \tag{22}$$

In his study, Jones B. [36] showed that the formula of the *K*-coefficient according to Mair R.J. et al. [33] leads to an overestimation of the parameter i_x in the case of deep tunnels. In his work, Tupikov M.M. [37] mainly considered the inside workings of communication tunnels by mechanized shield tunneling and concluded that the values of the excess excavated soil coefficient according to field data vary from 1.8 to 5.8 % for small shallow tunnels. The least squares approximation method was used to obtain the values of correction coefficients for the surface settlement formula (20) of Peck R.B. [1] and the formula for surface and structure settlements within the boundaries of relative depth of tunnels in the range of the ratio to diameter as follows H/D 1÷2,5.

Table 1

Summary table of the values for determining the maximum surface settlement (S_{max}) of various researchers

Таблица 1

Сводная таблица значений определения максимальных осадок поверхности (*S*_{max}) различных исследователей

Author(s)	Field of application	Formula S _{max}						
Peck R.B. [1]	The first concept of the empirical method	$S_{v}(x) = S_{v,\max} \cdot e^{-\frac{x^{2}}{2t_{x}^{2}}}$						
O'Reilly M.P. & New B.N. [26]	Studies of shallow tunnels;	$S_{v,\max} = \frac{\pi \cdot V_L}{i_x} \cdot \frac{D^2}{4} = 0,313 \cdot \frac{V_L \cdot D^2}{i_x}$						
Attewell P.B. et al. [27]	field measurement data	$S_{\text{max}} = \frac{A_l V_L}{100} \frac{1}{\sqrt{2\pi L_{\text{inf}}}}$, where $L_{\text{inf}} = k_I z_0$						
Herzog M. [28]	A series of experimental studies of known results	$S_{max} = 0.785(\gamma z_0 + \sigma_s) \left(\frac{D^2}{i_x E}\right)$						
Sagaseta C. [6]	Shallow tunnelling studies;	$S_{max} = \frac{V_L}{2\pi z_0} \left(1 + \frac{y}{y^2 + z_0^2} \right)$						
Rankin W.J. [30]	experimental studies	$S_{max} = 0.0125 V_L \left(\frac{R^2}{i_x}\right)$						
	A series of experimental studies of known results according	$S_{max} = 0.0125 K \left(\frac{R^2}{i_x} \right)$, where $K = V_L$						
Arioglu E. [32]	to Rankın W.J. (1988) [46]; field measurement data	$K = 0.87e^{0.26N} = 0.87e^{0.26\left(\frac{\gamma_{0} + \sigma_{s} - \sigma_{T}}{c_{u}}\right)}$						
	A series of experimental studies of known results according to Herzog M. (1985) [21], but for 2-tunnels	$S_{max} = 4.71(\gamma z_0 + \sigma_s) \left(\frac{D^2}{(3i_x + a)E}\right)$						
Lee C.J. et al. [34]	A series of test experimental studies	$\left(\frac{S_{max}}{D}\right) = 0.00398 \left(\frac{z_0}{D}\right)^{-0.58} \cdot V_L(\%)$						
Möller S.C. [35]	Comparison with known results; field measurement data	$S_{v,\max} \approx \frac{A_t}{i\sqrt{2\pi}} \cdot GLR$, where $GLR = \frac{V_t}{A_t} \approx \frac{V_s}{A_t}$						
Tupikov M.M. [37]	A series of test experimental studies; studies of known results; field meas- urement data	$S_{v}(y) = C_{1}S_{v,\max}e^{\frac{C_{2}y^{2}}{2t_{x}^{2}}}, \text{ where:}$ $C_{1}(\chi) = 1,525 - 1,147 \cdot \chi + 0,353 \cdot \chi^{2}$ $C_{2}(\chi) = 1,23 - 0,871 \cdot \chi + 0,212 \cdot \chi^{2} \text{ where:}$ $\chi = z_{0} / D \text{ for the range of } 1 \div 2,5$						
Protosenya A.G. et al. [39]	Comparison with known results; field measurement data; empirical studies	$S_{v,\max} = \frac{2\pi R U_r}{i_x \sqrt{2\pi}}$, where $U_r = \frac{R}{2G} (\gamma H - \sigma_T)$						
Chakeri H. & Ünver B. [38]	Comparison with known results; field measurement data	$S_{\max} = 3198.744 \left(\frac{D}{z_0}\right) \cdot \left(\left(\frac{\gamma z_0 + \sigma_s - (C + 0.3\sigma_T)}{E}\right)(1 - \upsilon)(1 - \sin\varphi)\right)^{0.8361}$						

Remark: parameters for S_{max} determination by different researchers: k_I – coefficient of the Gaussian function determining the position of the trough inflection point of the soil surface settlement; A_t – Tunnel cross-sectional area, m²; γ – specific gravity of the soil, kN/m³; σ_s – additional surface load, kPa; E – modulus of soil deformation, kPa; σ_T – bottom-hole face loading pressure, kPa; c_u – shear strength in undrained state, kPa; a – distance between tunnels (with closed tunnelling), m. U_r – radial displacement of the tunnel contour, m; G – shear modulus of the soil mass, MPa.

Chakeri H. & Ünver B. [38] proposed a new equation for determining the soil deformations at the surface and a formula for determining the distance to trough bending point (i_x) of the Gaussian distribution curve. In their work Protosenya A.G. et al. [39] during development of a method for predicting the bottom-hole loading pressure and soil surface settlement in TBM tunneling the land surface settlement itself is considered similarly to Möller S.C. [35] proceeding from the material balance of the "lost volume" V_{sl} and the volume of soil surface settlement V_s .

All the main dependencies of empirical methods for determining day surface settlement have been summarised in a single Table 1. For ease of reading, different researchers' designations of formula parameters, which are identical in meaning, are brought to a uniform format at the authors' discretion.

Tables 2–5 of the parameters required to calculate soil surface settlement (S_{max}) according to the formulae of various researchers are presented below.

Table 2

Values of parameters *K*' and *n* from the type of the soil according to Clough G.W. & Schmidt B. [25]

Таблица 2

Значения параметров *K*' и *n* от типа грунта по Clough G.W. & Schmidt B. [25]

Type of the soil	K'	п
for clays	1.0	0.8÷1.0
for wet granular soil	0,74	0.9
for dry granular soil	0.63	0.97

Table 3

Values of the parameter *K* from the type of the soil according to O'Reilly M.P. & New B.N. [26]

Таблица 3

Значения параметра *K* от типа грунта по O'Reilly M.P. & New B.N. [26]

Type of the soil	K
Technogenic (filled soil)	0.2
Sands coarse to dusty, dense to medium density	0.3
Sands grail and dusty, loose sands	0.2
Clay layers and clayish soil semi-solid and quasi-plastic	0.4
Clay layers and clayish soil soft-, quasi-liquid and liquid soil (possibly with organic)	0.7

Table 4

Values of the parameter k_I from the type of the soil according to Attewell P.B. et al. [27]

Таблица 4

Значения параметра k_I от типа грунта по Attewell P.B. et al. [27]

Type of soil or rock	k _I
Cohesive soils	0.3
Normally compacted clays	0.5
Overcompacted clays	0.6–0.7
Shale	0.6–0.8
quartz rock	0.8-0.9

Table 5

Values of excess excavation ratio V_L according to Attewell P.B. et al. [27]

Таблица 5

Значения коэффициента перебора V_L по Attewell P.B. et al. [27]

Tunnel technology	V_L
With the use of a mechanized tunneling complex with the opening of the tunnel for the entire section	0.5–1.0
With step-by-step opening of the tunnel section	0.8-1.5

All the main dependencies of empirical methods for determining the distance to the inflection point of the settlement curve (i_x) have been summarised in a single Table 6. For ease of reading, different researchers' designations of formula parameters which are identical in meaning have been brought to a uniform format at the discretion of the authors.

Table 6

Summary table of values for determining the distance to the inflection point of the settlement curve (i_x) of different researchers

Таблица 6

Author(s)	Field of application	Formula <i>i</i> _x
Knothe S. [20]	All types of soils (mining method)	$i_x = \frac{z_0}{\sqrt{2\pi} \cdot \tan\left(45^\circ - \frac{\varphi}{2}\right)}$
Peck R.B. [1]	All types of soils	$\frac{i_x}{R} = \left(\frac{z_0}{2R}\right)^n : \text{ where } n = 0, 8 - 1, 0$
Schmidt B. [40]	All types of soils	$i_x = R \left(\frac{z_0}{2R}\right)^{0.8}$
Cording E.J. & Hansmire W.H. [21]	All types of soils (mining method)	$i_x = R \sec \psi + (C+R) \tan \psi,$ где $\psi = \left(45^\circ - \frac{\varphi}{2}\right)$
Attewell P.B. [41]	All types of soils	$\frac{i_x}{R} = \alpha \left(\frac{z_0}{2R}\right)^n$: where $\alpha = 1,0$ и $n = 1,0$
Atkinson J.H. &	Loose sands	$i_x = 0.25(z_0 + R)$
Potts D.M. [22]	Compact sands and clays	$i_x = 0.25 (1.5z_0 + 0.5R)$
Glossop N.H. [23]	Coehesive soils	$i_x = 0.5 z_0$
Clough G.W. & Schmidt B. [25]	Clayer soils	$i_x = K' \cdot \frac{D}{2} \cdot \left(\frac{z_0}{D}\right)^n (K' - \text{Table 1})$
Mair R.J. <i>et al</i> . [24]	All types of soils	$i_x = 0.5 \cdot z_0$
	Coehesive soils	$i_x = 0.43z_0 + 1.1$ at $3 \le z_0 \le 34$
O'Reilly M.P. & New B N [26]	Non-Coehesive soils	$i_x = 0.28z_0 - 0.1$ at $6 \le z_0 \le 10$
	All types of soils	$i_x = K \cdot z_0 \ (K - \text{Table 2})$

Сводная таблица значений определения расстояния до точки перегиба кривой осадок (*i_x*) различных исследователей

The end of Table 6

Окончание табл. 6

Author(s)	Field of application	Formula i_x							
	Coehesive soils	$i_x = 0.57 + 0.45(z_0 - z) \pm 1.01$							
Leach G. [29]	Non-Coehesive soils	$i_x = 0.64 + 0.48(z_0 - z) \pm 0.91$							
Herzog M. [28]	All types of soils	$i_x = 0.4z_0 + 1.92$							
	Clayer soils	$i_x = 0.4z_0 + 1.6$							
Ariagh E [22]	All types of soils	$i_x = 0.38z_0 + 2.84$							
Allogiu E. [52]	All types of soils, shield tunnelling only	$i_x = 0.9R \left(\frac{z_0}{D}\right)^{0.88}$							
Merkezi Y. [42]	All types of soils	$\frac{2i_x}{D} = 1.392 \left(\frac{z_0}{D}\right)^{0.704}$							
		$i_x = K(z_0 - z^*);$							
Mair R.J. <i>et al</i> . [33]	All types of soils	$K = \frac{0.175 + 0.325(1 - z^* / z_0)}{1 - z^* / z_0}$							
Jones B. [36]		$K = -0.25 \ln (z_0 - z) + 1.234$							
Loganathan N. & Poulos H.G. [10]	All types of soils	$\frac{i_x}{R} = 1.15 \left(\frac{z_0}{2R}\right)^{0.9}$							
Lee C.J. et al. [34]	All types of soils	$i_x = 0.29 \left(1 - \frac{z^*}{z_0} \right) z_0 + R$							
Han X. [43]	All types of soils	$i_x = (1 - 0.02\varphi) z_0$							
Wei G. [44]	All types of soils	$i_x = m \left(R + z_0 \tan\left(45^\circ - \frac{\varphi}{2}\right) \right)$							
Протосеня А.Г. и др. [39]	All types of soils	$i_x \cong R + z_0 \cdot \tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)$							
Chakeri H. & Ünver B. [38]	All types of soils	$i_x = 0.6054(0.87z_0 + 0.13D) - 2.8562$							
Zhu B. <i>et al</i> . [45]	All types of soils	$i_x = 0.51z_0 + 0.48$							

Remark: *z – distance from the surface to the subsurface (the depth of the foundation L_p).

Methods and methodology

In the course of a comprehensive comparative and content analysis of various approaches to determining soil surface settlement during tunnelling operations, which constitutes the adopted research method, a slight downward trend in the number of papers on analytical and empirical calculation methods worldwide has been observed.

The paper includes a basic classification of methods for calculating day surface settlement and determining the distance to the inflection point of the settlement curve (i_x) . A historical selection of analytical and empirical methods is made in chronological order of the development of data analyses and theoretic items. This approach considers a variety of sources from different years to provide a broader overview of tunnelling data. Some comparative research results are

analysed in the next paragraph. Finally, a discussion and conclusions are offered with the main objective of performing an overview study on numerical methods with classification and results.

Results

The analytical and empirical methods for determining settlement and described in the first paragraph have a number of limitations and simplifications, such as: no consideration of complex geological conditions (non-horizontal layering, hydrogeological conditions, physical and mechanical characteristics of the soil mass), no consideration of the real spatial rigidity of structures and their current technical condition, as well as the presence of defects, etc., which in some cases even excludes the possibility of using the method.

Among the past studies in the the present article it has been reviewed the work of Melis M. & Rodriguez J.M. [46, 47], in which a comparative analysis of analytical and empirical methods with numerical ones is carried out for the tunnel borings of the Madrid Metro Extension and compared with the results of field measurements (Fig. 6).

	Numerical model			Peck			Oteo			Loganathan-Poulos			Verruijt-Booker			Sagaseta			Measured data		
Section	δ _{max} (mm)	i (m)	V _S (%)	δ _{max} (mm)	i (m)	V _S (%)	δ_{max} (mm)	i (m)	V _S (%)	δ _{max} (mm)	i (m)	V _S (%)	δ _{max} (mm)	i (m)	V _S (%)	δ _{max} (mm)	<i>i</i> (m)	V _S (%)	δ _{max} (mm)	i (m)	V _S (%)
I	-7.7	6.0	0.12	-32.3	6.2	0.50	-6.2	6.2	0.10	-11.0	7.5	0.23	-15.0	7.0	0.34	-12.8	7.0	0.25	-4.6	8.0	0.39
II	-17.2	7.4	0.32	-22.1	6.3	0.35	-6.8	6.3	0.11	-10.4	9.0	0.22	-10.0	8.5	0.24	-8.6	8.0	0.18	1.9	12.0	0.12
III	-9.9	9.5	0.26	-11.1	8.3	0.23	-7.9	8.3	0.16	-9.0	7.0	0.23	-5.5	6.5	0.16	-4.6	6.5	0.12	-4.4	8.0	0.13
IV	-16.4	3.5	0.18	-56.6	4.9	0.69	-11.5	4.9	0.14	-14.5	5.0	0.24	-26.8	4.5	0.48	-22.7	4.3	0.35	-5.6	6.0	0.15
V	-6.6	10.0	0.24	-9.5	10.5	0.25	-5.3	10.5	0.14	-7.9	10.5	0.22	-5.3	8.0	0.17	-4.5	7.5	0.13	1.4	11.0	-0.03

Fig. 6. Calculated and measured values of settlement (δ_{max}), distance to the inflection point of the settlement trough (*i*) and outbreak ration (V_S) corresponding to the analysed sections Рис. 6. Расчетные и измеренные значения осадок (δ_{max}), расстояния до точки перегиба мульды осадок (*i*) и коэффициента перебора грунта (V_S), соответствующие анализируемым участкам

In his study A.M. Hein [14] compares the analytical method developed by him – the reciprocity theorem, with other analytical, empirical and numerical methods for driving two tunnels (Fig. 7).



Fig. 7. Comparison of all settlement profiles in the process of driving two tunnels for the assessment of surface settlement according to the work of A.M. Hein [14] Рис. 7. Сравнение всех профилей осадок при проходке двух тоннелей для оценки осадок поверхности по работе А.М. Хейн [14]

Of the relatively recent papers on empirical methods, Chakeri H. & Ünver B. [38], where the authors, having analysed the results of calculations and monitoring of surface settlement and based on the work of Herzog M. [28] proposed a new empirical equation for determining the soil settlement on the surface. The researchers note the significant factors affecting the maximum surface settlement, which are: tunnel diameter, tunnel depth, specific gravity, internal friction angle, cohesion, Young's Modulus, Poisson's ratio, pressure in the digging face of the tunnel-boring complex and surface load. It can be concluded from these factors that the authors understand and highlight the importance of physical and mechanical characteristics of the boring soil mass, which are the "basic" nature of the engineering survey data.

In the work of Guskov I.A. et al. [48], it is carried out a comparative analysis of the analytical method of Y.A. Limanov [4] and the numerical method using the FEM calculation with the PC "Plaxis" (Fig. 8).



Fig. 8. Results of analytical and numerical methods according to Guskov I.A. et al. [47] Рис. 8. Результаты аналитического и численного методов по Гуськову И.А. и др. [47]

In general, it can be noted that the empirical equations of the following researchers are more popular in practical application and convergence of results: O'Reilly M.P. & New B.M. [26]; Attewell P.B. et al. [27]; Rankin W.J. [30] and Mair R.J. et al. [33] with additions by Jones B. (2010) [24]; and as a comparative version with the generally recognised study by Peck R.B. [1]. According to modern requirements early works of analytical methods are not sufficiently accurate and more recent works, such as those by Loganathan N. & Poulos H.G. [10], are sometimes characterised by increased complexity of toric computations.

In this study we did not consider in detail such analytical and empirical methods as the determination of the settlement of overlying structures taking into account their reduced rigidity since with the development of numerical methods of FEM and FDM there is no direct need for this problem nowadays, and all studies of the last 15-20 years were carried out for the characteristic concept of day surface settlement. It should be noted that Burland J.B. & Wroth C.P. were the first to propose a method for determining the settlement of overlying structures with tensile stresses taking into account their reduced rigidity [49]. They considered the building as an ideal beam with length L and height H deforming under the central point load with maximum deflection Δ . It is argued that for structures subjected to deformations of tunnel penetrations, the restraining effect of the foundation will, in fact, result in a lowering of the neutral axis, which may therefore coincide with the bottom edge of the "beam". It is also concluded that the neutral axis remains in the middle of the "beam" and it is shown that this variant of the neutral axis location is consistent with observations of the operational characteristics of the structures. The expressions linking the ratio Δ/L for beam element with maximum bending strain (ε_b) and diagonal deformation (ε_d). The deformations of the structure with maximum settlement Δ can also be determined from the expressions of Burland J.B. at el. [50] in the form:

$$\frac{\Delta}{L} = \left\{ \frac{L}{12t} + \frac{3I \cdot E}{2tL \cdot H \cdot G} \right\} \varepsilon_b;$$
(23)

and

$$\frac{\Delta}{L} = \left\{ 1 + \frac{HL^2 \cdot G}{18 \cdot I \cdot E} \right\} \varepsilon_d ; \qquad (24)$$

where H – structure height; L – the length of the structure (but limited by any point of deflection or degree of settlement); E and G – reduced modulus of elasticity (Young's) and shear modulus for the beam element, respectively; I – moment of inertia of the beam element (i.e. $H^{3}/12$ in the deflection zone and $H^{3}/3$ in the holding zone)); t – the greatest distance from the neutral axis to the edge of the beam (i.e. H/2 in the deflection zone and H in the holding zone).

Later Potts D.M. & Addenbrooke T.I. [51, 52] used about 100 FEM models with nonlinear elastoplastic soil models in different tunnel configurations and structure sizes and introduced 2 important dimensionless coefficients: relative flexural stiffness (ρ^*), which expresses the relative stiffness between the structure and the soil; and relative axial stiffness (α^*), determined as:

$$\rho^* = \frac{EI}{E_s H^4}; \tag{25}$$

and

$$\alpha^* = \frac{EA}{E_S H}; \tag{26}$$

where H – half-width of the structure (= B/2); El and EA – flexural and axial stiffness of the structure; E_s – averaged modulus of deformation (Young) of the soil.

This is discussed in detail in Mair R.J. & Taylor R.N. [53], as well as in later works on modernisation of these methods, including for deep and pile foundations: Franzius J.N. & Addenbrooke T.I. [54]; Franzius J.N. et al. [55]; Farrell R. [56]; Mair R.J. [57]; Goh K.H. & Mair R.J. [58, 59]; Mair R.J. & Williamson M.G. [60]; Losacco N. et al. [61]; Giardina G. et al. [62]; Fargnoli V. et al. [63] and a general review on this topic in Franza A. [64].

Discussion and conclusion

An extensive review analysis of analytical and empirical methods for determining soil surface and structures settlement during tunnelling works allows us to conclude that these methods can no longer always meet all the conditions, approaches and standards of modern design methods for geotechnical and tunnelling problems in general, despite the relative simplicity, speed and engineering accuracy of soil surface settlement estimation. This thesis is also confirmed by the fact that for more than 30 years, the formulas for calculating settlement and calculating the distance of the inflection point (i_x) of the Gaussian settlement curve of the distribution of these methods have only been revised and supplemented by various researchers, and, in particular, with the use of regional design experience and field measurement data, but did not have a fundamentally new approach.

Based on this, it can be concluded that the absolute majority of modern research is reduced to the use of numerical methods for calculating the settlement of soil surface and structures, since

they most accurately meet the requirements of modern geomechanical models of the soil environment, taking into account all kinds of factors in assessing the impact of tunnel boring operations, such as: the ability to take into account complex geological conditions (non-horizontal layering and hydrogeological conditions of the massif, physical and mechanical characteristics of the mass soils), modeling of the real spatial stiffness of structures and their current technical condition, even with respect to defects, etc.

Taking into consideration the multitude of proposed methods and their variations by different authors who carried out their research in different soil conditions, it can be said that all empirical formulae are rather subjective and can be applied to a greater extent for a specific location. In conclusion, it should be mentioned that the analysis of long-term studies of different times and for various soil and technical conditions gives a good idea of the engineering thought development and accumulation of statistical material in such field of geotechnics as tunnelling, but its further development is seen in numerical modelling in search of optimal and qualitative solutions. It goes without saying that it should be utilized semicentennial experience of analytical and empirical methods for determining the settlement from tunnelling operations which has resulted in the construction of a huge number of existing tunnels of different types.

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