The task of this research is to determine the values of the generalized strength parameters of the foundation soil $\sigma^*_{\text{sw}}$ and $\sigma^*_{\text{se}}$, at which the depth $\Delta Z$ of the development of Coulomb areas of plastic deformation under the edges of the foundation of finite rigidity thickness $H$, loaded with a uniformly distributed load of variable intensity $q$ and foundation of the same thickness, bearing a rigid above-foundation structure of variable height $H^*$ will correspond to the closure of plastic area under the base of the foundation what is complied with the ultimate state of the foundation (according to Prandtl). To carry out computer modeling a list of variables of design parameters that influence the process of formation and development of plastic areas under the sole of the foundation, and the intervals of their change, has been established. As a result of the calculations and processing of the data obtained it was found that the numerical values of $\sigma^*_{\text{sw}}$ and $\sigma^*_{\text{se}}$ differ significantly from each other: equations of approximating curves of dependencies of the form $\sigma^*_{\text{sw}} = f\left(\frac{q}{\gamma H}\right)$ and $\sigma^*_{\text{se}} = f\left(\ln\left(\frac{E}{\varepsilon}\right)\right)$ and $\sigma^*_{\text{sw}} = f\left(\ln\left(\frac{E}{\varepsilon}\right)\right)$ have different forms correspondingly and are described by the different approximating expressions. If it is given the values of the generalized strength parameters of the foundation soil $\sigma^*_{\text{sw}}$ and $\sigma^*_{\text{se}}$, which the soil mass should have after its fixing, then using the graphs shown in Fig. 6–10 and the table it will be possible to conclude whether the limit state of the fixed soil base will be...
achieved under given loads. In other words, focusing on the numerical values of $\sigma^{*}_{sw}$ and $\sigma^{*}_{s}$, it is possible to determine the values $\gamma;\phi; c; E_o$, which must be obtained in the process of fixing the soil base so that the specified external load does not exceed the maximum permissible value.

Introduction

When determining the bearing capacity of the foundations loaded with a central vertical load the solution and design scheme proposed by L. Prandtl are considered as the main and classical solution [1].

Solving the problem of the theory of ideal plasticity about the indentation of a rigid die into a half-space, in the case of plane deformation, and using the condition of plasticity of Tresk – Saint-Venant, Prandtl determined the depth of the focus of large plastic deformations, the extent of deformation zones on the free surface, the stress state in the plastic region, contact stresses and the force of insertion of the punch into the half-space in the absence of contact friction.
Later R. Hill [2] showed that L. Prandtl’s solution is not the only one and proposed another slip field. The corresponding calculation scheme has also found application in soil mechanics: both solutions give the same expressions for the value of the normal stress on the contact surface.

However, historically, in the vast majority of cases the methods for calculating the bearing capacity of foundations are based on this scheme, therefore, presenting the material we’ll rely on it.

If a normal uniformly distributed load is applied to the surface of the soil the intensity of which is increased gradually, then the soil under load, while its intensity is not high, will be compacted until local shifts occur under the edges of the load at points A and C (Fig. 1), that is, a state in which the Coulomb strength condition [3, 4] is fulfilled (limit state). The resultant of this load is called the $P_{ult}$ first critical load. The expression for its finding was first given by N.P. Puzyrevsky [5]

$$P_{ult}^1 = \frac{\pi (\gamma h + c \cdot \text{ctg} \phi)}{\text{ctg} \phi + \phi - \pi / 2} + \gamma h. \quad (1)$$

It is considered that if the areas of plastic deformations are developed under the foundation base to a depth of $\Delta z \leq \frac{b}{4}$, then the base is linearly deformable, and the load value corresponding to the fulfillment of the condition $\Delta z = \frac{d}{4}$ is called the calculated resistance

$$R = \frac{\pi (0.25\gamma b + \gamma h + c \cdot \text{ctg} \phi)}{\text{ctg} \phi + \phi - \pi / 2} + \gamma h. \quad (2)$$

If the load is $P \leq R$, then the settlement of the foundation can be calculated in the framework of linear elasticity.

The analysis of formula (2) indicates that the value of $R$ calculated with its use can be increased significantly with increasing the width of the foundation plate $b$ (included in the numerator of expression (2)), which is pointed out by A.V. Pilyagin [6, 7]. The fact that the value of the designed resistance of the foundation $R$, calculated by formula (5.7) “Set of Rules 22.13330.2016. Foundations of buildings and structures” has highly overestimated values is also mentioned in [8–10].

With a further increase of the load the bearing capacity of the soil is completely exhausted, accompanied by the closure of areas of plastic deformations (at point D in Fig. 1) and the completion of the formation of an elastic soil core ($\Delta A D C$, Fig. 1), which force aside the soil to the left and right of itself, resulted in global deformations of the base. The load corresponding to this moment is called the second critical or maximum permissible load.

For the first time, the formula for determining the magnitude of this load for a weightless base being under the influence of a uniformly distributed band load of intensity $q$ was obtained by L. Prandtl [1] and X. Reisner [11]:
Considering the formula (3) we see that in no way is it taken into account the rigidity of the foundation, the numerical values of the Poisson coefficients \( \mu \) (coefficients of lateral pressure \( \xi_o \)) of the soil and the foundation material, its width, thickness, depth of laying, and so on. Probably due to this fact many theoretical [12–14] and experimental studies [15–17] indicate overestimated values of the \( P_{ult} \) obtained by the formula (3).

Indeed, it is impossible to get the analytical solution of determining \( i_{\text{ult}}^2 \), which would take into account all the above mentioned factors. So, numerical methods come to the rescue. In particular, in the works [18–20] it was investigated the problems of formation and development of plastic deformation areas in the homogeneous base, taking into account the rigidity of the foundation and the above-foundation structure. It was noted that areas of plastic deformations under the stamp thickness \( H \) begin to form not only under its edges, but also in the depth of the core under its base. As the numerical value of the ratio of the deformation modulus of the stamp material and the base soil \( E/E_o \) increases, the shape of the compacted soil core (CSC) arising under the stamp changes from the shape of a curved trapezoid to the shape of a triangle with curved sides.

The process of development of the plastic deformations area (PDA) under a thick stamp with all the considered values of \( E/E_o \) begins under its edges, and the shape of the soil core in the form of a triangle with a curved boundary remains constant. In this case, the closure of plastic areas occurs at approximately the same depth. The values of the maximum permissible load and design resistance for a thick stamp (\( H^* = 6H \), consideration of the rigidity of the above-foundation structure) are 17–25.9 % and 2.8–24.3 % higher than the corresponding values for a stamp with a thickness of \( H \) loaded with a uniformly distributed load equivalent in force with the numerical values of the \( E/E_o \) ratio considered in the work.

The goal of the research

With reference to the above mentioned investigations we specify the problem of determining the value of the given cohesion pressure of the foundation base which should be achieved in the process of the foundation base reinforcement irrespective of the method used and when the base does not undergo structural failure. In other words, it is necessary to determine numerical values of the generalized strength parameters of the foundation soil \( \sigma_{\text{cm}}^{\text{ext}} \) and \( \sigma_{\text{cm}}^{\text{int}} \) provided that:

\[
\frac{E}{E_o} = 1; 2; 5; 10; 100; 1000; \quad \frac{b}{H} = 2; 4; 6; 10; 20.
\]

The intensity of the uniformly distributed load transmitted to a rigid foundation with a thickness of \( H \) consistently takes the values \( q(\gamma_o H)^{-1} = 0; 3; 12; 20 \), the angle of internal friction of the soil \( \varphi = 20^\circ, 25^\circ, 30^\circ, 35^\circ \), and the height of the rigid above-foundation part of the structure is consistently assigned the values \( H^* = 0, 1.5H, 6H, 10H \) (considering the conditions [19–21], that \( \gamma = 2\gamma_o \), it is not difficult to see that the corresponding values of the uniformly distributed load and the load from the basement part of the structure (\( H^* \)), in the power sense, are equivalent).
The value of the reduced cohesion pressure (4) is taken as a generalizing strength parameter due to the fact that (see Fig. 2) the manifestation of soil cohesion seems to be equivalent to a fictitious increase of the normal stress in the shear plane what increases the strength of the soil

\[ \sigma_{ca} = c (\gamma \tan \varphi)^{-1}, \]  

where: \( H \) – foundation depth; \( c, \gamma, \varphi \) – adhesion, specific gravity and soil cohesion.

All calculations have been performed with the help of computer programs [22, 23], in which it is formalized the finite element method, allowing to take into account most of the parameters listed in the introduction influencing the process of development of Coulomb plastic deformation regions (CDRs).

In the process of computer modeling the situation when according to the Prandtl calculation scheme (Fig. 1) the PDAs closing takes place is assumed as the moment of the limit state appearing.

**Results of calculation**

Having regard to the conditions of the set task and with the help of computer programs [22, 23] it has been obtained 960 numerical values of generalized strength parameters of the foundation soil \( \sigma_{ca}^{in} \) and \( \sigma_{ca}^{out} \) each, corresponding to all possible combinations of design variables, the limits of variation of which are given above.

As an example Fig. 3 shows the plastic deformation areas in the base of the foundation with footing depth and thickness \( H \) and width \( b = 6H \) at \( E/E_a = 10 \) with the depth of the rigid above-foundation part \( H^* = 1.5H; 6H; 10H \) (a–c) and equivalent in terms of force uniform intensity load \( q = 3\gamma H; 12\gamma H; 20\gamma H \) (d–f).

Analysis of the figures shows that the areas of plastic deformation under the stamp of thickness \( H \) develop not only under its edges but also in the depth of the core under its foot. The vertical cross-section of the elastic core (EC) has the shape of a curvilinear trapezoid. The process of PDAs development under a thick (height \( H^* \)) stamp starts under its edges, and the vertical cross-section of the soil core has the form of an isosceles triangle with curvilinear sides. The closing of the plastic areas occurs at approximately the same depth. The corresponding values of the numerical values of the generalized strength parameters of the foundation soil \( \sigma_{ca}^{in} \) and \( \sigma_{ca}^{out} \) for a thick stamp (thickness \( H^* \), taking into account the rigidity of the above-foundation structure) and a stamp with thickness \( H \) (rigidity of the above-foundation part is not taken into account) may differ by 20–140 % from each other.
Fig. 3. Plastic deformation areas in the foundation base of thickness and footing depth $H$ and width $b = 6H$ at $E/E_o = 10$ with the depth of the rigid above-foundation part $H^* = 1.5H; 6H; 10H$ (a–c) and equivalent in terms of force uniform intensity load $q = 3\gamma H; 12\gamma H; 20\gamma H$ (d–f)
Fig. 4. Graphical dependences of the form $\sigma_{cn}^{in} = f\left(\frac{q}{\gamma H}\right)$ and $\sigma_{cs}^{in} = f\left(H^*\right)$ at $b = 2H; 4H; 6H$ and $E/E_0 = 10$ without regard to (a–c) and with regard to (d–f) rigidity of the above-foundation structure.
Fig. 4 shows graphical dependences of the form $\sigma_{ca}^{\text{max}} = f(q / \gamma H)$ and $\sigma_{ca}^{\text{max}^*} = f(H^*)$ at $2b = 2H; 4i; 6H$ and $E/E_o = 10$ without regard to (a–c) and with regard to (d–f) rigidity of the above-foundation structure, plotted on the basis of the calculations, in the process of which the areas of plastic deformations, given in Fig. 3 are obtained. The analysis of the pictures shows that dependences of the form $\sigma_{ca}^{\text{max}} = f(q / \gamma H)$ with certainty $R^2 = 1$ are approximated by direct lines. The curves of the form $\sigma_{ca}^{\text{max}^*} = f(H^*)$ with certainty $R^2 = 0.99–1.0$ are approximated by logarithmic curves.

It should be mentioned that a linear approximation of the dependences in the form of $\sigma_{ca}^{\text{max}^*} = f(H^*)$ can also be carried out; then the accuracy of the approximation will be 10–35 % lower. The exception is curves $\sigma_{ca}^{\text{max}^*} = f(H^*)$ at $2b = 2H$, the accuracy of approximation of which by straight lines is equal to $R^2 = 1$.

![Graphical dependences](image)

Fig. 5. Graphical dependences of the form $\sigma_{ca}^{\text{max}} = f(2b) \quad \text{and} \quad \sigma_{ca}^{\text{max}^*} = f(2b)$ at $\varphi = 25^\circ$ and $E/E_o = 10$ without regard to (a) and with regard to (b) rigidity of the above-foundation structure

Fig. 5 shows graphical dependences of the form $\sigma_{ca}^{\text{max}} = f(2b) \quad \text{and} \quad \sigma_{ca}^{\text{max}^*} = f(2b)$ at $\varphi = 25^\circ$ and $E/E_o = 10$ without regard to (a) and with regard to (b) the rigidity of the above-foundation structure, from which it can be seen that in both cases the value $2b$ affects the required strength properties of the soil and the nature of this influence (the shape of the corresponding curves) is similar in both cases. Difference of numerical values of quantities $\sigma_{ca}^{\text{max}}$ and $\sigma_{ca}^{\text{max}^*}$ can be up to 20 % depending on the value of parameter $2b$.

Fig. 6–10 show graphical dependences of the form $\sigma_{ca}^{\text{max}^*} = f\left(\ln\left(\frac{E}{E_o}\right)\right)$ and $\sigma_{ca}^{\text{max}} = f\left(\ln\left(\frac{E}{E_o}\right)\right)$ to determine the required values of generalized strength parameters with regard to (a–c; $H^* = 1,5H; 6H; 10H$ correspondingly) and without regard to (d–f; $q = 3\gamma H; 12\gamma H; 20\gamma H$ correspondingly) of the above-foundation structure rigidity in case of $2b = 2H; 4H; 6H; 10H; 20H$. From these figures it can be seen that the curves of the form $\sigma_{ca}^{\text{max}^*} = f\left(\ln\left(\frac{E}{E_o}\right)\right)$ are approximated by straight lines while the curves of the form $\sigma_{ca}^{\text{max}} = f\left(\ln\left(\frac{E}{E_o}\right)\right)$ – by polynomials of the third degree, moreover, the accuracy of the approximation in both cases is $R^2 \geq 0.97$. 

99
Fig. 6. Graphical dependences of the form \( \sigma_{cb}^{\text{max}} = f^* \left( \ln \left( \frac{E}{E_0} \right) \right) \) and \( \sigma_{cb}^{\text{max}} = f \left( \ln \left( \frac{E}{E_0} \right) \right) \) for determining the maximum permissible load with regard to (\( a-c; H^* = 1.5H; 6H; 10H \) correspondingly) and without regard to (\( d-f; q = 3\gamma H; 2\gamma H; 20\gamma H \) correspondingly) rigidity of the above-foundation structure at \( 2b = 2H \).

Fig. 7. Graphical dependences of the form \( \sigma_{cb}^{\text{max}} = f^* \left( \ln \left( \frac{E}{E_0} \right) \right) \) and \( \sigma_{cb}^{\text{max}} = f \left( \ln \left( \frac{E}{E_0} \right) \right) \) for determining the maximum permissible load with regard to (\( a-c; H^* = 1.5H; 6H; 10H \) correspondingly) and without regard to (\( d-f; q = 3\gamma H; 12\gamma H; 20\gamma H \) correspondingly) rigidity of the above-foundation structure at \( 2b = 4H \).
Fig. 8. Graphical dependences of the form $\sigma_{cs} = f^*(\ln \left( \frac{E}{E_0} \right))$ and $\sigma_{ct} = f^*(\ln \left( \frac{E}{E_0} \right))$ for determining the maximum permissible load with regard to ($a$–$c$; $H^* = 1.5H$; $6H$; $10H$ correspondingly) and without regard to ($d$–$f$; $q = 3\gamma H$; $12\gamma H$; $20\gamma H$ correspondingly) rigidity of the above-foundation structure at $2b = 6H$.

Fig. 9. Graphical dependences of the form $\sigma_{cs} = f^*(\ln \left( \frac{E}{E_0} \right))$ and $\sigma_{ct} = f^*(\ln \left( \frac{E}{E_0} \right))$ for determining the maximum permissible load with regard to ($a$–$c$; $H^* = 1.5H$; $6H$; $10H$ correspondingly) and without regard to ($d$–$f$; $q = 3\gamma H$; $12\gamma H$; $20\gamma H$ correspondingly) rigidity of the above-foundation structure at $2b = 10H$. 
Fig. 10. Graphical dependences of the form $\sigma_{ca} = f\left(\ln \left(\frac{E}{E_o}\right)\right)$ and $\sigma_{ca} = f\left(\ln \left(\frac{E}{E_o}\right)\right)$ for determining the maximum permissible load with regard to (a–c; $H^* = 1.5H; 6H; 10H$ correspondingly) and without regard to (d–f; $q = 3\gamma H; 12\gamma H; 20\gamma H$ correspondingly) rigidity of the above-foundation structure at $2b = 20H$.

The graphical interpretations of the calculation results shown in Fig. 3–10 indicate that the numerical values of the generalized strength parameters of the foundation soil $\sigma_{ca}^{\text{th}}$ and $\sigma_{ca}^{\text{th}}$ for the thick stamp (thickness of $H^*$, consideration of rigidity of the above-foundation structure) and stamp of $H$ thickness (rigidity of the above-foundation part is not taken into account) depend significantly on all the factors considered in this paper: the width of the foundation $2b$, the force equivalent parameters $q$ and $H^*$, the ratio of deformation modules $E/E_o$ and the angle of the internal friction of the foundation soil $\phi$. The absence of some curves on these or that graphs suggests that the numerical values of variables of the calculated parameters reflected in the graphs, the limiting state of the base (the closure of plastic areas under the foundation base) is not achieved even with $\sigma_{ca}^{\text{th}} = \sigma_{ca}^{\text{th}} = 0$ (as it is known $\sigma_{ca}^{\text{th}} \geq 0$ и $\sigma_{ca}^{\text{th}} \geq 0$, which follows from the formula (4)).

**Conclusion**

In the process of designing buildings and structures raised on fixed (reinforced) foundations numerical values of the required quantities of the generalized strength parameters of foundation soil $\sigma_{ca}^{\text{th}}$ and $\sigma_{ca}^{\text{th}}$ can be determined according to the graphs given in Fig. 6–10 and constructed on the base of the results of multivariate computer analysis. To find $\sigma_{ca}^{\text{th}}$ and $\sigma_{ca}^{\text{th}}$ for intermediate values of variables of calculated parameters interpolation methods should additionally be used.
The tables show the numerical values of the coefficients for the approximating curves which determine the numerical values of $\sigma_{ca}^{ax}$ (see Fig. 6–10, a–c) for foundations with a rigid part above the foundation height $H^*$. Using interpolation methods it is possible to determine these coefficients for specific values of variables of the design parameters and calculate the corresponding value of $\sigma_{ca}^{ax}$. In the same way it is possible to calculate the required value $\sigma_{ca}^{ax}$, if we take the numerical values of the corresponding coefficients directly from the Fig. 6–10, d–f.

**Values of coefficients of approximating curves**

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It should be mentioned that the results obtained by means of the above graphs and tables are in qualitative agreement with the data of the papers [23–25].

**Финансирование.** Исследование не имело спонсорской поддержки.

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Библиографический список

5. Пузыревский Н.П. Теория напряженности землистых грунтов. – Л.: Ленинград. ин-т инженеров путей сообщения, 1929. – 66 с.


22. FEA: свидетельство о государственной регистрации программы для ЭВМ № 2015617889 / Богомолова О.А. и др.; зарег. 23 июля 2015 г.


References

7. Pilyagin A.V. To the question of determining the design resistance of foundations under different loading schemes. Izvestiya KGASA, 2004, no. 1 (2), pp. 43-44.


