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О НЕЛИНЕЙНЫХ ИНТЕГРАЛЬНО-ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЯХ ВОЛЬТЕРРА – ФРЕДГОЛЬМА

Рассмотрены две нелинейные задачи в терминах абстрактных операторных уравнений вида $Bx = f$. В первой задаче оператор B содержит линейный дифференциальный оператора A, оператор Вольтерра K с ядром сверточного типа и скалярное произведение векторов $g(x)\Phi(u)$ с нелинейными граничными функционалами Ф. Первая задача записывается в виде уравнения $Bu(x) = Au(x) - Ku(x) - g(x)\Phi(u) = f(x)$ при граничных условиях $D(B) = D(A)$. Во второй задаче оператор B содержит линейный дифференциальный оператора A и скалярное произведение векторов $g(x)\mathrm{F}(Au)$ с нелинейными ограниченными на $C[a, b]$ функционалами F, где $F(Au)$ обозначают нелинейный интеграл Фредгольма. Вторая задача задается уравнением $Bu = Au - gF(Au) = f$ при граничных условиях $D(B) = D(A)$.

Предложен прямой метод поиска точных решений нелинейных интегрально-дифференциальных уравнений Вольтерра – Фредгольма, а именно в настоящей работе доказаны три теоремы о существовании точных решений.

Первая теорема означает, что при ненулевой константе α_0 интегрально-дифференциальное уравнение Вольтерра $Au(x) - Ku(x) = 0$ сводится к интегральному уравнению Вольтерра и имеет уникальное нулевое решение. В то же время оператор $A - K$ замкнутый и непрерывно обратимый. Также если функции $u(t)$, $g(t)$ и $f(t)$ имеют экспоненциальный порядок α, то неоднородное уравнение $Au(x) - Ku(x) = f(x)$ для каждой $f(x)$ имеет уникальное решение, показанное в настоящей работе.

Вторая теорема означает, что для первой исследуемой задачи с обратимым оператором $A - K$, для $f(x)$ и $g(x)$, принадлежащих непрерывному отрезку $[a, b]$, точное решение определяется уравнением $u = (A - K)^{-1}f + (A - K)^{-1}g\mathbf{b}^*$ для каждого вектора $\mathbf{b}^* = \Phi(u)$, который решает нелинейную алгебраическую (трансцендентную) систему из n уравнений $\mathbf{b} = \Phi((A - K)^{-1}f + (A - K)^{-1}g\mathbf{b})$. В случае если последняя алгебраическая система не имеет решения, то исследуемая задача также не имеет решения.

Третья теорема означает, что точное решение второй исследуемой задачи определяется уравнением $u = A^{-1}(f+g\mathrm{d}^*)$ для каждого вектора $\mathbf{d}^* = F(Au)$, который решает нелинейную алгебраическую (трансцендентную) систему из n уравнений $\mathbf{d} = F(f + g\mathbf{d})$. В этом случае мы имеем такое же свойство, если последняя алгебраическая система не имеет решения, то исследуемая задача также не имеет решения

Два частных примера рассмотрены для каждой исследуемой задачи, показывающие получение точных решений путем применения предложенного метода. В первом примере рассмотрено интегрально-дифференциальное уравнение Вольтерра – Фредгольма, а во втором примере рассмотрено уравнение с нелинейным интегралом Фредгольма.

Ключевые слова: краевые задачи, начальные задачи, теоремы существования решения, нелинейные интегрально-дифференциальные уравнения, уравнения Вольтерра – Фредгольма, нелинейный интеграл Фредгольма, точные решения, прямой метод, оператор Вольтерра сверточного типа, абстрактные операторные уравнения.

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ABOUT NONLINEAR INTEGRO-DIFFERENTIAL VOLTERRA AND FREDHOLM EQUATIONS

Two nonlinear problems in terms of abstract operator equations of the form $Bx = f$ are investigated in this paper. In the first problem the operator B contains a linear differential operator A, the Volterra operator K with kernel of convolution type and the inner product of vectors $g(x)\Phi(u)$ with nonlinear bounded functionals Φ. The first problem is given by equation $Bu(x) = Au(x) – Ku(x) – g(x)\Phi(u) = f(x)$ with boundary condition $D(B) = D(A)$. In the second problem the operator B contains a linear differential operator A and the inner product of vectors $g(x)\mathrm{F}(Au)$ with nonlinear bounded on $C[a, b]$ functionals F, where $F(Au)$ denotes the nonlinear Fredholm integral. The second problem is presented by equation $Bu = Au – gF(Au) = f$ with boundary condition $D(B) = D(A)$.

A direct method for exact solutions of nonlinear integro-differential Volterra and Fredholm equations is proposed. Specifically, the three theorems about existing exact solutions are proved in this paper.

The first theorem is mean that for nonzero constant α_0 Volterra integro-differential equation $Au(x) – Ku(x) = 0$ is reducing to Volterra integral equation and has a unique zero solution. During it the operator $A – K$ is closed and continuously invertible. Also, if the functions $u(t)$, $g(t)$ and $f(t)$ are of exponential order α then nonhomogeneous equation $Au(x) – Ku(x) = f(x)$ for each $f(x)$ has a unique solution, shown in this paper.

The second theorem is mean that for the first investigated problem with an injective operator $A – K$, for $f(x)$ and $g(x)$ from $C[a, b]$, the exact solution is given by equation $u = (A – K)^{-1}f + (A – K)^{-1}g\mathbf{b}^*$ for every vector $\mathbf{b}^* = \Phi(u)$ that solves nonlinear algebraic (transcendental) system of n equations $\mathbf{b} = \Phi((A – K)^{-1}f + (A – K)^{-1}g\mathbf{b})$. And if the last algebraic system has no solution, then investigated problem also has no solution.

The third theorem is means that exact solution of the second investigated problem is given by $u = A^{-1}(f+g\mathrm{d}^*)$ for every vector $\mathbf{d}^* = F(Au)$ that solves nonlinear algebraic (transcendental) system of n equations $\mathbf{d} = F(f + g\mathbf{d})$. In this case we have same property – if the last algebraic system has no solution, then investigated problem also has no solution.

Two particular examples for each considered problem are shown for illustration of exact solutions giving by perform the suggested in this paper methods. In the first example was considered integro-differential Volterra and Fredholm equation and in the second case was considered equation with nonlinear Fredholm integral.

Keywords: Boundary value problems, initial problems, Existence Theorem, nonlinear integro-differential equation, Volterra and Fredholm equations, Fredholm integral, exact solutions, direct method, Volterra operator with kernel of convolution type, abstract operator equations.

Introduction

The Volterra and Fredholm integro-differential equations (IDEs) appear in modeling many situations in areas such as mechanics, electromagnetic theory, population dynamics, pharmacokinetic studies, forestry and many others [1–6]. Existence and Uniqueness Theorems are illustrated for the systems of Volterra IDEs of first order in [7–10]. The finding for exact solutions to nonlinear integro-differential equations is a difficult problem, because this problem is reduced to a nonlinear system of algebraic (transcendental) equations. The solutions of nonlinear Volterra and Fredholm

IDEs are obtained in the most cases by numerical methods [11, 12]. Exact solutions of Volterra and Fredholm integro-differential equations where studied in [4], [13–21]. In this paper we investigate two problems with equation $Bx = f$. In the first problem the operator B contains a linear differential operator A, the Volterra operator K with kernel of convolution type and the inner product of vectors $g(x)\Phi(u)$ with nonlinear bounded functionals $\Phi_i, \quad i = 1,\dots,n,$ i.e the next problem

$$Bu(x) = Au(x) - Ku(x) - g(x)\Phi(u) = f(x), D(B) = D(A), \qquad (1)$$

where

$$Au(x) = \sum_{i=0}^{n} \alpha_i u^{(n-i)}(x), \quad D(A) = \left\{u(x) \in C^n[a,b] : u^{(j)}(a) = 0\right\},$$

$$Ku(x) = \sum_{j=0}^{n} \int_a^x K_j(x-t) u^{(j)}(t)\,dt, \quad K_j(x) \in C[a,b],$$

$$\Phi(u) = col\big(\Phi_1(u),\dots,\Phi_n(u)\big), \quad \Phi_k(u) = \int_a^b q_k(t)\left[u^{(k-1)}(t)\right]^s dt,$$

$$g(x) = \big(g_1(x),\dots g_n(x)\big), \quad g_k(x), q_k(x) \in C[a,b],$$

$$\alpha_0 \neq 0, k = 1,\dots,n, \quad j = 0,1,\dots,n, \quad s \in R.$$

In the second problem the operator B contains a linear differential operator A and the inner product of vectors $g(x)F(Au)$ with nonlinear bounded on $C[a,b]$ functionals F_i, $i = 1,\dots,n,$ where $F(Au)$ denotes the nonlinear Fredholm integral.

This paper is the continuation of the paper [16], and the generalization of the papers [18, 21], where

$$Bu = \hat{A}u - g\Phi(u) = f(x), \quad D(B) = D(\hat{A}), \quad D(\hat{A}) \subseteq Z \subseteq X,$$

$\hat{A}$ is a linear correct operator, X, Z are Banach spaces and Φ_i are linear bounded functionals on Z in [18], or Φ_i are nonlinear bounded functionals on Z, $i = 1,\dots,n$ in [21]. Note that, usually, a nonlinear Fredholm integro-differential problem is not correct because it is transformed to nonlinear algebraic (transcendental) equation which has most from one solutions.

1. Terminology and notation

Let X, Y be complex Banach spaces and X^* is an adjoint to X space, i.e. the set of all complex-valued linear and bounded functionals f on X. We denote by $f(x)$ the value of f on x. We write D(A) and R(A) for the domain and the range of the operator A, respectively. A linear operator B is said to be an injective, if $\ker B = 0$. An operator $A: X \to Y$ is called invertible if there exists the inverse operator A^{-1}. An operator $A: X \to Y$ is called continuously invertible if it is invertible and the inverse operator A^{-1} is continuous on Y. Remind that every linear injective operator is invertible. A function $f(x)$ is said to be of exponential order α if $\left|f(x)\right| \le Me^{\alpha x}, 0 \le x < \infty,$ where M, α are constants. Everywhere in this paper we use the notations $u = u(x), g = g(x), f = f(x)$. Remind that a linear operator $A: X \to Y$ is called correct if $R(A) = Y$ and the inverse A^{-1} exists and is bounded on Y. Denote

$$C_0[a,b] = \{u(x) \in C[a,b] : u(a) = 0\},$$

$$C_0^n[a,b] = \left\{u(x) \in C^n[a,b] : u^{(j-1)}(a) = 0, \quad j = 1,\ldots,n\right\},$$

$$\hat{K}(x-t) = \frac{1}{\alpha_0}\left(K_{0,n} + K_{1,n-1} + K_{2,n-2} + K_{3,n-3} + \ldots + K_{n,0}\right)(x-t)$$
$$-\frac{1}{\alpha_0}\left[\alpha_1 + \alpha_2(x-t) + \ldots + \frac{\alpha_n}{(n-1)!}(x-t)^{n-1}\right],$$

where $K_{j,i+1}(z)$ is the antiderivative of $K_{j,i}(z)$, $j = 0,1,\ldots,n, i = 0,1,\ldots,n-1, K_{j,0}(z) = K_j(z),$ and $K_j(0) = 0$ or $K_j'(0) = 0,$ the index i in $K_{j,i}(z)$ shows the number of integrations of the function $K_j(z)$. From Theorems 1, 2 [17] immediately follows the next theorem.

Theorem 1. *Let* $\alpha_0 \ne 0$. *Then*

(i) Volterra integro-differentizal equation $Au(x) - Ku(x) = 0$ *or*

$$\sum_{i=0}^{n}\alpha_i u^{(n-i)}(x)-\sum_{j=0}^{n}\int_a^x K_j(x-t)u^{(j)}(t)dt=0,\quad u(x)\in C_0^n[a,b], \tag{2}$$

is reduces to Volterra integral equation

$$u(x)-\int_a^x \hat{K}(x-t)u(t)=0, \tag{3}$$

and has a unique zero solution.

(ii) The operator A-K is closed and continuously invertible, i.e. $R(A-K)=C[a,b]$ *and* $(A-K)^{-1}$ *is continuous on* $C[a,b]$.

(iii) If the functions $u(t), g(t), f(t)$ *are of exponential order* α *then nonhomogeneous equation*

$$Au(x)-Ku(x)=f(x)$$

for each $f(x)$ *has a unique solution*

$$u(x)=L^{-1}\left\{\frac{F(s)}{\alpha_0 s^n+\alpha_1 s^{n-1}+\ldots+\alpha_n-K(s)}\right\},\quad x\in[a,+\infty), \tag{4}$$

where $\mathrm{F}(s)=L\{f(x)\},\quad \mathrm{K}(s)=L\{K(x)\}\neq\alpha_0 s^n+\alpha_1 s^{n-1}+\ldots+\alpha_n$, *where L is Laplace Transform and* L^{-1} *is Inverse Laplace Transform.*

We prove the next theorem.

Theorem 2. *Let the operators* $B,A,K:C[a,b]\to C[a,b]$, *the vectors* g,Φ *be defined as in (1) and A-K be an injective operator. Then:*

(i) For $f(x),g(x)\in C[a,b]$ *the exact solution to the problem (1) is given by*

$$u=(A-K)^{-1}f+(A-K)^{-1}g\mathrm{b}^*, \tag{5}$$

for every vector $\mathrm{b}^*=\Phi(u)$ *that solves nonlinear algebraic (transcendental) system of n equations*

$$\mathrm{b}=\Phi\left((A-K)^{-1}f+(A-K)^{-1}g\mathrm{b}\right), \tag{6}$$

(ii) If (6) has no solution, then (1) also has no solution.

Proof. (i), (ii). Since A − K is a linear injective operator and by Theorem 1, $R(A-K)=C[a,b]$ then from (1) we get

$$u=(A-K)^{-1}g\Phi(u)+(A-K)^{-1}f. \tag{7}$$

Acting by functional vector Φ on both sides of (7), we obtain

$$\Phi(u)=\Phi\Big((A-K)^{-1}g\Phi(u)+(A-K)^{-1}f\Big). \tag{8}$$

Denoting by b = Φ(u) and substituting in (8) we arrive to the nonlinear algebraic (transcendental) system of n equations

$$\mathbf{b}=\Phi\Big((A-K)^{-1}g\mathbf{b}+(A-K)^{-1}f\Big). \tag{9}$$

Let b∗ be a solution of this system satisfying b∗ = Φ(u). By substitution of b∗ into Equation (7) we obtain the solution (5). It is evident that if (6) has no solution, then (1) also has no solution. The theorem is proved.

Remark. If the initial conditions in (1) are nonhomogeneous, i.e.
$u^{(j)}(a)=\alpha_j,\quad \alpha_j\in R, j=0,1,\ldots,n-1,$ then by transformation

$$v(x)=u(x)-\sum_{k=0}^{n-1}\frac{\alpha_k}{k!}(x-a)^k$$

the problem (1) is reduced to the case with homogeneous initial conditions and then we can apply Theorem 2.

The next theorem is useful to solve the nonlinear Fredholm IDEs.

Theorem 3. *Let the operators B, A: $C[a,b]\to C[a,b]$, A is a linear correctoperator,*

$$Bu=Au-gF(Au)=f,\quad D(B)=D(A), \tag{10}$$

and the vectors $g=(g_1,\ldots,g_n)$, $F=col(F_1,\ldots,F_n)$, $g_i\in C[a,b]$, F_i nonlinear bounded functionals on $C[a,b]$, $i=1,\ldots,n$. Then:

(i) For $f \in C[a,b]$ *the exact solution of Problem (10) is given by*

$$u = A^{-1}\left(f + g\mathbf{d}^*\right), \tag{11}$$

for every vector $\mathbf{d}^* = F(Au)$ *that solves nonlinear algebraic (transcendental) system of n equations*

$$\mathbf{d} = F\left(f + g\mathbf{d}\right), \tag{12}$$

(ii) If (12) has no solution, then Problem (10) also has no solutions.

Proof. (i), (ii). From (10), since A is a linear correct operator, we obtain

$$Au = gF(Au) + f, \tag{13}$$

$$u = A^{-1}gF(Au) + A^{-1}f. \tag{14}$$

Acting by functional vector F on both sides of (13) we obtain

$$F(Au) = F\left(f + gF(Au)\right). \tag{15}$$

Denoting by $\mathbf{d} = F(Au)$ and substituting in (15) we arrive to the nonlinear algebraic (transcendental) system of n equations

$$\mathbf{d} = F\left(f + g\mathbf{d}\right).$$

Let $\mathbf{d}^*$ be a solution of this system satisfying $\mathbf{d}^* = F(Au)$. By substitution of $\mathbf{d}^*$ into Equation (14) we obtain (11). It is evident that if (12) has no solution, then (10) also has no solution. The theorem is proved.

Below we give two examples which show the usefulness of our results.

Example 1. The next nonlinear integro-differential Volterra and Fredholm equation

$$\begin{gathered} u'(x) + 3\int_0^x \cos(x-t)u(t)\,dt - \cos x\int_0^{\pi} u^2(t)\,dt = \frac{3x}{\pi}\sin x, \\ u(0) = 0, \quad u(x) \in C^1[0,\pi], \end{gathered} \tag{16}$$

has two exact solutions

$$u_1(x) = \frac{2}{\pi}\sin x, \quad u_2(x) = \frac{2}{\pi}\left(2\sin 2x + \sin x\right). \tag{17}$$

Proof. If we compare the equation (16) with (1) it is natural to take

$$Au(x)=u'(x),\quad Ku(x)=-3\int_0^x \cos(x-t)u(t)dt,\quad \Phi(u)=\int_0^\pi u^2(t)\,dt,$$
$$Bu=Au-Ku-g\Phi(u),\quad D(B)=D(A)=\{u(x)\in C^1[0,\pi]:u(0)=0\},$$
$$g(x)=\cos x,\quad f(x)=\frac{3x}{\pi}\sin x.$$

We can use Theorem 2. Let L,L^{-1} be the operators of the direct and inverse Laplace transform, respectively. Denote by $L[u(x)]=U(s)$ and $L[y(x)]=Y(s)$). The functions $x\sin x,\cos x$ are continuous on each closed interval $[0,b],\ \ b<\infty$. Furthermore $x\sin x$ is of exponential order 1 and $\cos x$ is of exponential order 0. So, we can use Laplace transform. Note that every solution of (16) on $[0,\infty)$ is also the solution of (16) on $[0,\pi]$.

From

$$(A-K)u(x)=u'(x)+3\int_0^x \cos(x-t)u(t)\,dt=y(x),\quad u(0)=0,$$

by using the Laplace transform and convolutions operator we get

$$sU(s)+\frac{3s}{s^2+1}U(s)=Y(s) \text{ or } U(s)=\left(\frac{3s}{4(s^2+4)}+\frac{1}{4s}\right)Y(s).$$

Now by using the inverse Laplace transform we obtain

$$u(x)=\frac{1}{4}(1+3\cos 2x)*y(x) \text{ or } (A-K)^{-1}y(x)=\frac{1}{4}\int_0^x[1+3\cos 2(x-t)]y(t)\,dt.$$

Then

$$(A-K)^{-1}\mathbf{b}g(x)+(A-K)^{-1}f(x)=$$
$$=\frac{1}{4}\int_0^x[1+3\cos 2(x-t)]\left(\mathbf{b}\cos t+\frac{3t}{\pi}\sin t\right)dt=\frac{(\pi\mathbf{b}-2)\sin 2x}{2\pi}+\frac{2\sin x}{\pi},$$
$$\Phi\left((A-K)^{-1}\mathbf{b}g+(A-K)^{-1}f\right)=\int_0^\pi\left(\frac{(\pi\mathbf{b}-2)\sin 2x}{2\pi}+\frac{2\sin x}{\pi}\right)^2dx=$$
$$=\frac{1}{8\pi}\left(\pi^2\mathbf{b}^2-4\pi\mathbf{b}+20\right).$$

Now by using (6) we obtain

$$\mathbf{b} = \frac{1}{8\pi}\left(\pi^2\mathbf{b}^2 - 4\pi\mathbf{b} + 20\right).$$

Solving this equation, we find

$$\mathbf{b}_1^* = \frac{10}{\pi}, \quad \mathbf{b}_2^* = \frac{2}{\pi}.$$

Substituting these values into (5) we obtain (17).

The next nonlinear Fredholm IDE we solve by Theorem 3.

Example 2. Let the operator $B : C\left[0,1\right] \to C\left[0,1\right]$ be defined by

$$u''\left(x\right) - 24\sin x\int_0^{\pi}\left(t-\pi\right)\left[u''\left(t\right)\right]^2 dt = \left(6\pi^2 - 1\right)\sin x, \quad u(0) = 0, \quad u'(0) = 0. \tag{18}$$

Then Problem (18) has two exact real solutions

$$u_1\left(x\right) = \sin x - x, \quad u_2\left(x\right) = \frac{6\pi^2 - 1}{6\pi^2}\left(x - \sin x\right). \tag{19}$$

Proof. If we compare the equation (18) with (10) it is natural to take

$$Au\left(x\right) = u''\left(x\right), \quad D\left(A\right) = \left\{u \in C^2\left[0,1\right] : u\left(0\right) = 0, u'\left(0\right) = 0\right\},$$

$$g\left(x\right) = 24\sin x, \quad f\left(x\right) = \left(6\pi^2 - 1\right)\sin x, \quad F\left(Au\right) = \int_0^{\pi}\left(t-\pi\right)\left[u''\left(t\right)\right]^2 dt.$$

Then

$$Bu = u''\left(x\right) - 24\sin x\int_0^{\pi}\left(t-\pi\right)\left[u''\left(t\right)\right]^2 dt, \quad D\left(B\right) = D\left(A\right), \quad F\left(f\right) = \\ = \int_0^{\pi}\left(t-\pi\right)\left[f\left(t\right)\right]^2 dt$$

and

$$F\left(f + g\mathbf{d}\right) = \int_0^{\pi}\left(t-\pi\right)\left[\left(6\pi^2 - 1\right)\sin t + 24\mathbf{d}\sin t\right]^2 dt = \\ = \left(24\mathbf{d} + 6\pi^2 - 1\right)^2 \frac{\left(-\pi^2\right)}{4}. \tag{20}$$

From equation (12) and (20) we get $\mathbf{d} = -\left(24\mathbf{d} + 6\pi^2 - 1\right)^2 \frac{\pi^2}{4}$, Using Derive program we compute

$$\mathbf{d}_1^* = -\frac{\pi^2}{4}, \quad \mathbf{d}_2^* = -\frac{36\pi^4 - 12\pi^2 + 1}{144\pi^2}. \tag{21}$$

We remind that

$$A^{-1} f\left(x\right) = \int_0^x \left(x - t\right) f\left(t\right) dt. \tag{22}$$

Then substituting the values $\mathbf{d}_1^*, \mathbf{d}_2^*$ from (21) into (11) and using (22), we obtain

$$u_1\left(x\right) = A^{-1}\left(f + g\mathbf{d}_1^*\right) = \int_0^x \left(x - t\right)\left[\left(6\pi^2 - 1\right)\sin t - \frac{\pi^2}{4} 24 \sin t\right] dt = \sin x - x,$$

$$u_2\left(x\right) = A^{-1}\left(f + g\mathbf{d}_2^*\right) = \int_0^x \left(x - t\right)\left[\left(6\pi^2 - 1\right)\sin t - \frac{36\pi^4 - 12\pi^2 + 1}{144\pi^2} 24 \sin t\right] dt =$$

$$= \frac{6\pi^2 - 1}{6\pi^2}\left(x - \sin x\right),$$

which gives (19).

Исследование не имело спонсорской поддержки. Авторы заявляют об отсутствии конфликта интересов.

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