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Article / Статья

COMPARISON OF METHODS FOR PREDICTING THE SPATIAL DISTRIBUTION OF P_2O_5 IN THE ORE BODY OF THE APATITE CIRCUS DEPOSIT

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СРАВНЕНИЕ МЕТОДОВ ПРОГНОЗА ПРОСТРАНСТВЕННОГО РАСПРЕДЕЛЕНИЯ P_2O_5 В РУДНОМ ТЕЛЕ МЕСТОРОЖДЕНИЯ АПАТИТОВЫЙ ЦИРК

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In recent decades, there has been an active interaction of geological and mathematical sciences. One of the main directions of the introduction of mathematics into geology and the practice of geological exploration is the mathematical modeling of geological objects. In the Kirovsk-Apatit district of the Murmansk region, enterprise the Kirov branch of Apatit, JSC, is developing six fields: Plateau Rasvumchorr, Kukisvumchorrskoe, Yuksporskoe, Apatity circus, Koashviskoye and Nyurpakhskoye. At the moment, Ventyx MineScape (Australia) is actively being implemented in Apatit JSC. This is a set of integrated modules used in mining operations at enterprises conducting open / underground mining of ore deposits. Also at the Mining Institute of the Kola Scientific Center of the Russian Academy of Sciences (Apatity, Murmansk region), the computer modeling system MINEFRAME has been created and constantly improved. Today, it is an integrated software package designed to solve a wide range of geological, mining and technological problems. As example of the Apatite Circus deposit, the prospects of joint use of mining and geological information systems MineScape and MINEFRAME for mathematical modeling of geological objects and a geostatistical description of the spatial distribution of the mineral are shown. The article discusses the results of a geostatistical study of the distribution of the useful component (P_2O_5) within the ore body bounded by the framework model of balance ores from the Apatite circus deposit, and also two block models are constructed. The blocks of the first model were filled using the method of inverse distances, the blocks of the second - by the usual kriging method. At the end of the article, to select the most suitable method for the field, there has been made a comparison between the average contents obtained by using the methods of conventional kriging and inverse distances.

Ключевые слова:
Апатитовый цирк, полезный компонент, геостатистика, вариограмма, каркасная модель, блочная модель, интерполяция, метод обратного расстояния, кригинг.

В последние десятилетия наблюдается активное взаимодействие геологических и математических наук. Одно из главных направлений внедрения математики в геологию и практику геолого-разведочных работ – математическое моделирование геологических объектов. В Кировско-Апатитском районе Мурманской области градообразующее предприятие Кировский филиал АО «Апатит» ведет разработку шести месторождений: Плато Расвумчорр, Кукисвумчоррское, Юкспорское, Апатитовый цирк, Коашвиское и Ньюрпакхское. В данный момент в АО «Апатит» активно внедряется Ventyx MineScape (Австралия) – это набор интегрированных модулей, используемых при ведении горных работ на предприятиях, ведущих открытую/подземную отработку пластовых/рудных месторождений. Также в Горном институте Кольского научного центра РАН (г. Апатиты, Мурманская область) создана и постоянно совершенствуется система компьютерного моделирования объектов горной технологии MINEFRAME. На сегодня она представляет собой интегрированный пакет программ, предназначенный для решения широкого круга горно-геологических и горно-технологических задач. На примере месторождения Апатитовый цирк показана перспективность совместного использования горно-геологических информационных систем MineScape и MINEFRAME для математического моделирования геологических объектов и геостатистического описания пространственного распределения полезного ископаемого. В статье рассмотрены результаты геостатистического исследования распределения полезного компонента (P_2O_5) в пределах рудного тела, ограниченного каркасной моделью балансовых руд месторождения Апатитовый цирк, а также построены две блочные модели. Блоки первой модели были заполнены с помощью метода обратных расстояний, блоки второй – методом обычного кригинга. В конце статьи приводится сравнение средних содержаний, полученных в результате интерполяции методами обычного кригинга и обратных расстояний, для выбора наиболее подходящего метода для данного месторождения.

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Introduction

From the results of discrete observations a description of the spatial distribution of geological exploration information as well as the prediction of their values at points or areas of the studied space by the mathematical modeling can be made. Currently, both in international practice and in Russia, the following contents calculating methods are mainly used:

1) polygonal method;
2) deterministic models;
3) probabilistic models (geostatistical);
4) stochastic modeling.

The first method was used before the development of computer simulation methods. Evaluation of the contents was carried out by weighted average at height. Volumes estimation – is the sections areas measurement with the subsequent volumes calculating of basic geometric figures.

In deterministic models, it is assumed that the spatial variable is a non-random function of coordinates and depends on the location of measurement points. The method of interpolation of the actual data values at the points of observation determines the used mathematical model type.

The geostatistical model considers the studied object as a geometric field with a certain law of spatial variability and with a well-defined value of the object at each of its points. The studied parameter is considered as a point spatial variable with a number of characteristics. The anisotropy of the spatial variable is expressed by the different rate of change of its values in different directions [1–8].

Stochastic modeling allows estimating the spatial variability and uncertainty of the data and generating a set of equiprobable implementations based on the initial distribution [9–12].

At the Mining Institute of the Kola Scientific Center, a mining computer simulation system MINEFRAME has been created and is constantly being improved. Today it is an integrated software package designed to solve a wide range of geological, mining and technological problems. To find dependencies describing the useful component content variability, the geostatistics methods are used, as well as the interpolation by the inverse distances method, taking into account the anisotropy ellipsoid of the studied characteristics [13–18].

Using the capabilities of MINEFRAME, according to operational and detailed exploration data (more than 11,000 samples), as well as 2 % and 4 % frame models of the P_2O_5 spatial distribution of the Apatite circus field, developed by the underground method by the Kirov branch of Apatit, an assessment of the mineral content distribution by two methods: geostatistical and inverse distances was performed.

Geological structure of the Apatite circus deposit

The history of the study of the world's largest Khibinsky alkaline massif has more than 150 years. The interest in it is caused by the relative rarity of alkaline complexes, the presence of the largest deposits of strategic raw materials and outstanding mineral diversity (over 400 minerals, including more than 100 discovered here for the first time).

By type, the Khibiny massif refers to asymmetric concentrically zonal plutons, while, according to geophysical data, the depth of its lower boundary is 5–7 km. The massif is composed of nepheline syenites, foidolites. Ultrabasic relics are present in nepheline syenites [19–22]. The dike series of lamprophyres, alkaline basalts, and tinguanites is widely developed. Nepheline syenites of the Khibiny contain agpaitic minerals only as accessories. The foyaites (chibinites) of the outer zone and the foyaits of the core of the massif have weakly expressed (hidden) stratification, which, like the trachytoid nature of these rocks, agrees with the general structure of the massif and falls at angles of 30–40° to the center of the concentric structure [23–25].

Despite the relatively simple structure of the Khibiny massif, not only the genesis, but also the age ratio of these rock complexes still causes discussions. To explain the array concentric-zonal structure a variety of genetic schemes are proposed.

All Khibiny deposits are confined to the part of the Main Foidolitic Ring, which is near to the Rischororite, where they form three ore fields: the south-east, south-west and north (Fig. 1). The deposits within each of these fields have a similar structure [26–30].

The Apatite circus field belongs to the south-western ore field and located between the Ukspor and Plateau Rasvumchorr deposits (see Fig. 1) and is a 12-km ore body with the Kukisvumchorr deposit, all parts of which have the same type of geological structure. The length of the Apatite circus itself along the strike is 2.5 km. The horizontal thickness of the deposit increases from 10–50 to 150–200 m from the northwest to the southeastern flank. The dip angles gradually increase with depth from 15–20º (at the upper level of the reservoir) to 40–50º (at the depth).

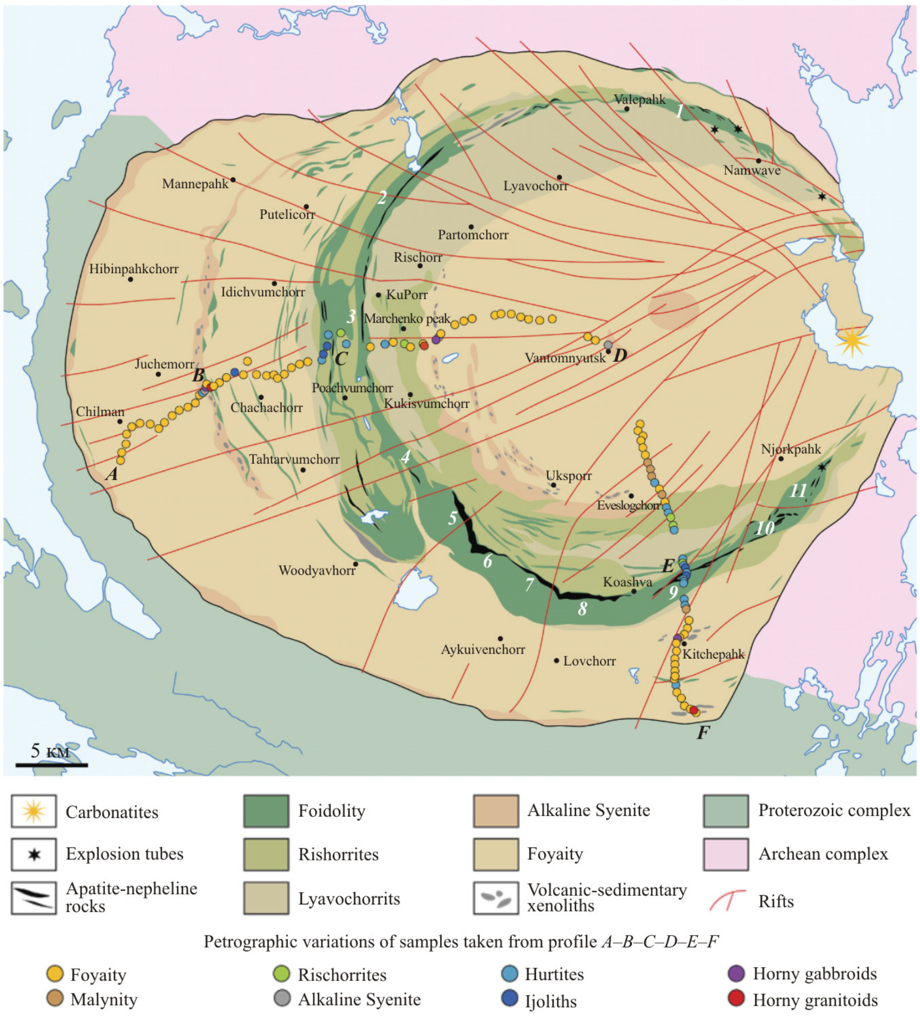

Fig. 1. Scheme of the Khibiny massif geological structure [32] with additions [26]. Apatite-nepheline deposits: *1* – Valepahk; *2* – Partomchorr; *3* – Quelporr; *4* – Snow Circus; *5* – Kukisvumchorr; *6* – Yukspor; *7* – Apatite circus; *8* – Plateau Rasvumchorr; *9* – Koashva, *10* – Njorkpahk; *11* – Oleniy Ruchey. *A* – *B* – *C* – *D* – *E* – *F* – profile with sampling points for studying of the array zonality

The most thick central part of the deposit is characterized by a zonal internal structure (in the direction from the lying to the hanging side): 1 – the zone of urtites with apatite impregnation (10–15 m); 2 – zone of mesh, lenticular-banded and block ores (50–100 m); 3 – zone of spotted and spotty-banded ores (25–75 m); 4 – discontinuous zone of titanite-apatite ores (20 m). The alternation of ore types is often disturbed by the appearance of the intracranial vein breccia layers in the different parts of the section [31, 32].

The field has been developing by the Rasvumchorr mine of the Kirov branch of Apatit JSC since 1954 by the underground method.

Geometry of the ore body of the Apatite circus deposit

Geometry of a deposit as one of the ways to interpret geological exploration data begins with the ore body delineating. This is a set of operations to allocate the volume of subsoil, which includes reserves that meet the condition. Contouring reflects the morphology, internal structure, occurrence conditions and ore bodies' continuity.

For the Apatite circus, the contours of the ore body were created in a two-dimensional space (AutoCad) by geologists of the Murmansk Geological Exploration Expedition JSC. The method of the reserves delineating for P_2O_5 cut-off grade, and thicknesses of ore bodies and empty interlayers included a number of operations: establishing, in accordance with the conditions, ore intervals by wells and mine workings (delineating by mining) and determining the contours of ore body reserves on the plan (delineating by area) 33–37].

Using the MineScape tools, the staff of the Kirov branch of Apatit JSC tied the ore body contours to the well trajectories in 3D space. Those wells that slightly deviate from the plane were also taken into account. Figure 2 shows a section along profile 9 + 00, in which it can be seen how in the recumbent side of the deposit, balance (4 % cut-off grade) ores turn into off-balance (2 % cut-off grade).

Skeleton modeling was performed by triangulation between contours in the MineScape environment. The first was built a 4 % frame model. The triangulation was carried out on the basis of the balance ores using tie lines playing the guides role. Then a 2 % skeleton was built, including balance and off-balance ores.

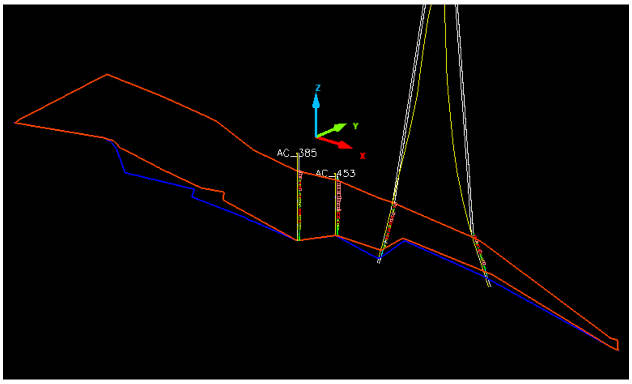

Fig. 2. 3D visualization of the ore body contours of the Apatite circus deposit along profile 9 + 00, MineScape

During the ore body's geometrisation, wells drilled from the surface at the preliminary exploration stage and specifying additional wells were taken into account.

Block model of the Apatite circus deposit ore body

The main task of exploration and subsequent study of mineral deposits is to obtain a reliable distribution model of the parameter being studied. To carry out interpolation and prediction of the main parameters of mineralization, it is necessary to create a geometrical basis – a block model.

A block model is an ordered set of rectangular parallelepipeds placed inside a closed frame surface. The parameters of the block model, determining the localization of its blocks in space, are the dimensions of the main block, the coordinates of the block center and the degree of its fragmentation. To better simulate the shape of the body near its boundary, smaller blocks can be used than the whole body.

In MINEFRAME, a block model was created by placing mini-blocks inside a 4 % frame model built in MineScape for interpolating P_2O_5 contents inside balance ores. The size of the blocks reflects the geometry, parameters of the ore deposit and takes into account the mining and technical requirements of mining. The size of the block model unit cells was dictated by the distances between the mining horizons, the density of the exploration network and the overall block model dimensions.

Based on the main exploration operational network of 50×50 m, the block size of 10×10 m was used at the Apatit circus field. The height of the block was taken to be equal to the substage height – 20–25 m. Subblocking (creating blocks with reduced dimensions at the contacts of the ore bodies) is used to more accurately calculate the ore volumes using the model. The subunit size in MINEFRAME is 5×5×7.5 m.

To interpolate the values of the useful component of the Apatite circus deposit two methods of mathematical modeling were chosen: the inverse distance method and kriging.

Inverse distance method

When using deterministic methods, it is assumed that the analyzed data is described by a certain deterministic function $V(x)$ defined on the studied region (D), where $x \in D$ is the coordinate of the point. The value at any point can be calculated on the basis of the interpolation function $V(x)$, constructed on the basis of the known data $V_i = V(x_i)$, measured at the points $xx_i \in D$.

The inverse distance method is a common method of the useful component content estimating of at a point. It is based on taking into account the distance from the cell to the nearest exploration workings. The content of the useful component in the estimated point is calculated by the formula [38]

$$C = \frac{\sum_{i=1}^{N} C_i d_i^{-m}}{\sum_{i=1}^{N} d_i^{-m}},$$

where C_i – is the content of the useful component in the sample; d_i – is the distance from the estimated point to the sample; N – is the number of samples; m – is the exponent (in our case, m = 2 is the inverse square distance method).

The contents of all samples that were included in the 4 % ore body skeleton of the Apatite circus deposit were interpolated into a block model. The assessment of the contents was carried out with composite samples – the sampling intervals were reduced to the same length, i.e. same statistical weight [38–41]. A length of 3 m was accepted as the most common sample length in a field testing database.

Figure 3 shows the block model section along the profile 9 + 00, in which the contents interpolation was carried out by the inverse distance method.

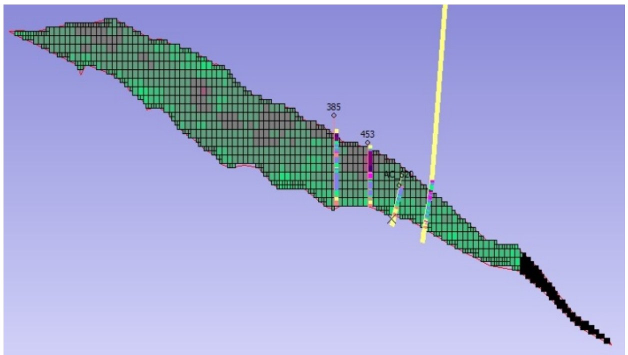

Fig. 3. Section of the ore body block model (inverse distance method) of the Apatite circus deposit along the profile 9 + 00, MINEFRAME

Kriging

The basic geostatistical estimation procedure is kriging. Method, named by J. Matheron after the geologist D.G. Krige, who studied the distribution of gold in the

South Africa deposits, gives the best effective estimate of values at unknown points or average values in blocks. There are several types of kriging: simple, normal, cooperative, indicator, universal, etc. To predict the distribution of P_2O_5 in the ore body of the Apatite circus deposit, the most commonly used method – ordinary kriging (hereafter referred to as OK) was used [42–49].

The idea of J. Matheron is to present the estimating function ξ^{OK} in the simplest way – in the form of a linear combination of values $\xi_1, \xi_2, \ldots, \xi_n$:

$$\xi^{OK} = \sum_{i=1}^{n} \lambda_i \xi_i.$$

If the true value of ξ were known, then one could give infinitely many such representations. This is the main advantage and difference of the method from many other approaches, in which the assessment of ξ is carried out through complex physical equations (diffusion, mass transfer, thermal conductivity, etc.), more or less approaching the actual process.

The problem is to choose the coefficients λ_i in conditions of uncertainty so that the estimate is effective. The latter involves two aspects: unbiasedness and minimum evaluation its dispersion [50–52].

The OK method takes into account not only the component distribution anisotropy, but also its statistical characteristics. When using the process, each structural or statistical domain is interpolated separately.

To determine the points (samples), when calculating the contents in each block, a search area is used, the parameters of which depend on the ore body morphology and the variogram model structure. In our case, the experimental general variogram (Fig. 4) is well approximated by the compositional model. Nuggets effect is visible, i.e. a jump from 0 to σ^2 when $h = 0$, and two spherical patterns with different angles of inclination. The parameters of the general variogram model are: the effect of the nuggets – 13.73; the threshold – 15.17–19.50; zone of influence, m – 18.00–105.00.

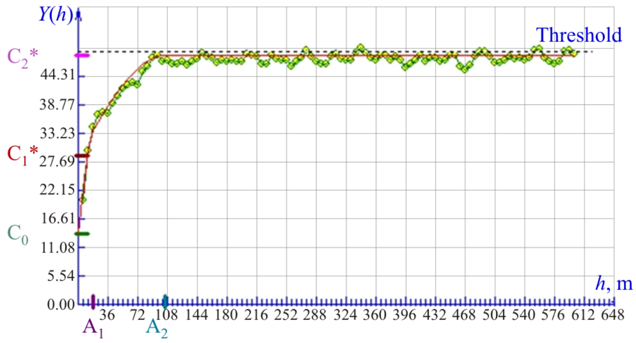

Fig. 4. General variogram of P_2O_5 distribution within the 4 % ore body of the Apatite circus deposit

The two main directions of the anisotropy in the ore body deposits were detected by variograms. The third direction is orthogonal to the first two ones. The first main direction is subparallel to the strike of the ore body. The second is consistent with the fall of the ore deposit. The revealed pattern is consistent with the geological feature of the deposit - a zonal structure (downwards) from rich ores to poor ones. Variograms are shown in Fig. 5.

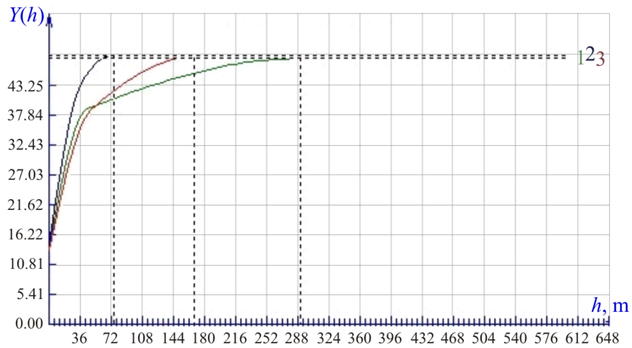

Fig. 5. Variograms in three anisotropy directions of the Apatite circus deposit ore body

The variograms along the directions are also three-structured, i.e. contain the nuggets effect and two spherical models. Options variograms are given in Table 1. The revealed anisotropy refers to a geometric type – models differ in zones of influence with equal thresholds (level of dispersion) [50–52].

Table 1

The parameters of the variogram model in the three main anisotropy directions of the Apatite circus deposit ore body

Direction	Azimuth, degree	Fall, degree	Nugget effect	Threshold	Influence zone, m
1	210	30	13.94	48.07	288.0
2	30	60	14.39	48.65	72.0
3	120	0	13.33	48.27	165.0

The obtained models allow us to justify the parameters of the search ellipsoid for contents interpolating into a block model.

The interpolation process was carried out sequentially with an increase in the radius of the search ellipsoid. If the block did not receive a rating of the content, the search ellipse increased until each block of the model received the predicted content.

Figure 6 shows a section of the block model along the profile 9 + 00, in which the interpolation of the contents was carried out using the ordinary kriging method.

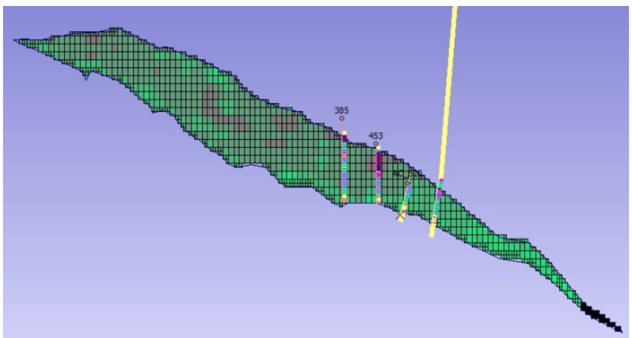

Fig. 6. Section of the block model (ordinary kriging method) of the Apatite circus deposit ore body along the profile 9 + 00, MINEFRAME

Conclusions

The main features of the block models are the possibility of interpolating the initial geological survey data into each block of the model. The block model reflects in detail the features of the ore body internal structure, regardless of the interpolation method used.

Table 2 presents a comparison of the average contents obtained as a result of interpolation using by ordinary kriging and inverse distances methods. Different methods of interpolation of contents in the block model give similar values, it indicates that selected data are correct.

Table 2

Comparison of the inverse distances interpolation methods and conventional kriging

Indicator	Model MINEFRAME	
	inverse distance method	ordinary kriging
Ore, thousand tons	105 081 393.33	105 081 397.25
P_2O_5, %	15.22	14.51

The inverse distance method has known drawbacks, since the interpolation is carried out without taking into account the spatial statistical regularities of the useful components distribution; also the source data density has a significant effect on the results. Since the Apatite Circus field has areas with the different exploration network density, it is advisable when applying the inverse distance method to select parameters for each section, which increases the simulation time.

Ordinary kriging has the following advantages as compared with the inverse distance method: before interpolating the contents, the statistical parameters of the useful components distribution are studied, which makes it possible to more fully reveal both the anisotropy parameters and the dependence of the change in the contents on the distance between the sampling points; In the calculations using the reverse kriging method, errors caused by the uneven density of the original data are significantly reduced.

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