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Study of Fluid Flow to a Horizontal Well

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Исследование течения флюида к горизонтальной скважине

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Recently, it is necessary to note the presence of negative dynamics in the deterioration of the reserves structure for newly discovered fields, and most of the them are classified as hard-to-recover, confined to deposits with a complex geological structure, low permeability, high oil viscosity, complicated by the presence of faults, active bottom waters and gas caps.
Hard-to-recover reserves are drilled with horizontal wells. This is primarily because horizontal wells make it possible to multiply the area of fluid filtration by increasing the drainage area due to the extensive contact of the horizontal well section with the rock, allowing to increase the well flow rate many times over.
Summarizing the above, horizontal wells are used to develop fields with the following parameters: fields with a thin oil-saturated rim (up to 15 m), with a gas cap and bottom water; fields of heavy oil, with a viscosity of more than 30 mPa·s; fields with low reservoir permeability (less than 0.002 μm2).
Under these conditions, linear Darcy's law cannot describe fluid filtration. Under the conditions of high-viscosity oil and low-permeability reservoir existence, a certain initial pressure gradient is determined, due to the rheological properties of the filtering fluid and high values of the surface friction coefficient. Under conditions of a thin oil rim and an increased gas factor, the limiting filtration rates due to the dissolved gas regime are observed, and a nonlinear law describes the fluid inflow.
One of the main parameters in the preparation of the technical and economic assessment of the reservoir is the flow rate of each individual horizontal well. Analytical methods for calculating the horizontal well flow rate show a high error. It is proposed to take a fresh look at the problem of determining the predicted flow rate of a horizontal well, using well-known approaches for solving this issue.
It is rather difficult to reliably predict the parameters of reservoir operation: the horizontal wells productivity obtained with the help of modern hydrodynamic stimulators turns out to be unreliable, which leads to the formation of an insufficiently rational development system. And the arising complications during operation in field conditions have to be eliminated due to significant volumes of material and labor resources. Thus, the development of methods that contribute to obtaining a reliable calculation of production is an urgent task for the oil industry.

Ключевые слова:
дебит горизонтальной скважины, линейная фильтрация, зона дренирования, трудноизвлекаемые месторождения, аналитические расчеты, приток флюида.

В последнее время необходимо отметить присутствие негативной динамики по ухудшению структуры запасов вновь открытых месторождений, и уже большая часть последних классифицируется как трудноизвлекаемые, приуроченные к залежам со сложным геологическим строением, низкой проницаемостью, высокой вязкостью нефти, осложненные наличием разломов, активных подошвенных вод и газовых шапок.
Разбуривание трудноизвлекаемых запасов месторождений происходит горизонтальными скважинами. Это обусловлено в первую очередь тем, что именно горизонтальные скважины позволяют многократно увеличить площадь фильтрации флюида за счет возрастания области дренирования, благодаря обширному контакту горизонтального участка скважины с породой, позволяя многократно увеличить дебит скважины.
Обобщая вышесказанное, горизонтальные скважины применяют для разработки месторождений со следующими параметрами: месторождения с тонкой нефтенасыщенной оторочкой (до 15 м), с газовой шапкой и подошвенной водой; месторождения тяжелой нефти, с вязкостью более 30 мПа·с; месторождения с низкой проницаемостью коллектора (менее 0,002 мкм^2).
В данных условиях фильтрация флюида не может быть описана линейным законом Дарси. В условиях существования высоковязкой нефти и низкопроницаемого коллектора определяется некий начальный градиент давления, обусловленный реологическими свойствами фильтрующейся жидкости и высокими значениями коэффициента поверхностного трения. В условиях тонкой нефтяной оторочки и повышенного газового фактора наблюдаются предельные скорости фильтрации за счет режима растворенного газа, и приток флюида описывается нелинейным законом.
Одним из основных параметров при составлении технико-экономической оценки залежи является дебит каждой отдельно взятой горизонтальной скважины. Аналитические методы расчета дебита горизонтальной скважины показывают высокую погрешность. Предлагается по-новому взглянуть на проблему определения прогнозного дебита горизонтальной скважины, используя известные подходы к решению данного вопроса.
Довольно затруднительно достоверно прогнозировать параметры эксплуатации залежей: производительность горизонтальных скважин, полученная при помощи современных гидродинамических стимуляторов, оказывается недостоверной, что в конечном итоге приводит к формированию недостаточно рациональной системы разработки, и возникающие осложнения при эксплуатации в промысловых условиях приходится устранять за счет значительных объемов материальных и трудовых ресурсов. Таким образом, разработка методик, способствующих получению достоверного расчета добычи, является актуальной задачей нефтедобывающей отрасли.

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Introduction

Simulation of fluid flows near horizontal wells is a challenging task. For a correct filtration process representation, first of all it is necessary to obtain formulas that adequately describe flow processes near a particular well.

D.W. Peaceman was closely involved in the construction of flow models in the near-wellbore area [1, 2]. In his works he assumed that the flow in the near-wellbore area was strictly radial, and introduced the concept of effective wellbore radius. The bottomhole pressure is replaced by the effective pressure, which corresponds to the effective radius. However, the works [1, 2] represent practical inapplicability of D.W. Peaceman's calculation procedure.

An interesting approach to the simulation of filtration near horizontal wells was proposed in the works of D.K. Babu [3, 4]. In these works, the issue was studied for the whole formation, and there was no tie to a well.

Both D. W Peaceman and D. K. Babu based their theories on the Darcy's linear filtration law, but this filtration law is applicable to a certain range of velocities.

The evaluation of horizontal wells productivity was carried out by such scientists as: A.M. Grigoryan [5], V.P. Pilatovsky [6–8], P.Ya. Polubarinova-Kochina [9], L.S. Leibenzon [10], V.P. Tabakov [11–14], Yu.P. Borisov [15], L.P. Stockman [16], F.M. Giger [17, 18], S.D. Joshi [19, 20], G.I. Renard, J. M. Dupuy [21, 22], D.K. Babu [3, 4], V.V. Sheremet [23] and others [24–45]. All works had the following assumptions:

1) the formation was considered isotropic (in some cases, vertical anisotropy was introduced);

2) the formation fluid was assumed to be incompressible with constant viscosity;

3) the fluid filtration follows the Darcy's linear filtration law;

4) the oil reservoir is a circular cylinder with constant height and natural drainage pattern;

5) the filtration mode is stationary;

6) hole friction is not taken into account.

To make it possible to separately account for the fluid inflow to estimate the same into a horizontal wellbore at a real field (with multiphase flotation), the authors use a correction for the relative permeability for each fluid separately.

Determination of the Flow Rate of Horizontal Well Symmetrically Located against the Formation Top and Bottom

Initially, Yu.P. Borisov and V.P. Tabakov [11–15] defined the following problem: it was required to determine the flow rate of a horizontal well of length L located in a formation of thickness h symmetrically against the formation top and bottom (Fig. 1), the external boundary radius R_{ef}, the external boundary pressure P_{ef}, with the formation absolute permeability k, the dynamic viscosity of the drained fluid μ, the bottomhole pressure P_w, the reduced radius of the well r_w. According to their research, the horizontal well flow rate is expressed by the formula:

$$q=\frac{2\pi kh}{\mu B}\cdot\frac{P_{ef}-P_w}{\ln\frac{4R_{ef}}{L}+\frac{h}{L}\ln\frac{h}{2\pi r_w}}. \qquad (1)$$

From the physical point of view, the first addend in the denominator reflects the external filtration resistance, the second one – the internal well resistance. This formula is

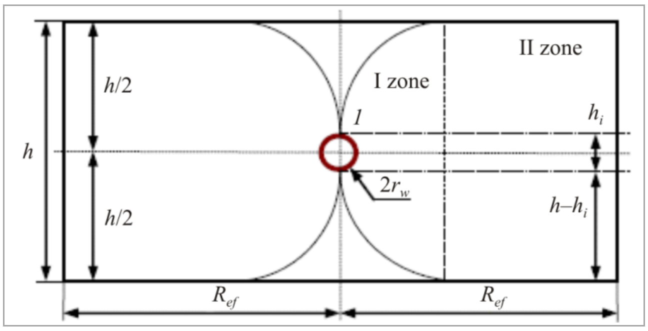

Fig. 1. Layout of a symmetrical horizontal wellbore by the formation thickness

based on the assumption that the horizontal well external boundary is considered to be circular and does not depend on its length.

Considering that F.M. Giger [17, 18] put forward an assumption that the horizontal well external boundary is elliptical rather than circular; he presented his formula for calculating the horizontal well flow rate as follows:

$$q=\frac{2\pi kh}{\mu B}\cdot\frac{P_{ef}-P_w}{\ln\frac{1+\sqrt{1-\left(L/2R_{ef}\right)^2}}{L/2R_{ef}}+\frac{h}{L}\ln\frac{h}{2\pi r_w}}, \qquad (2)$$

where R_k – the size of semi-major axis of an ellipse representing the external boundary.

S.D. Joshi [19, 20] assumed that there is a semi-major axis of an ellipse similar in area to a circle with a drainage radius R_{ef}, and obtained the following formula:

$$q=\frac{2\pi kh}{\mu B}\cdot\frac{P_{ef}-P_w}{\ln\frac{a+\sqrt{a^2-\left(L/2\right)^2}}{L/2}+\frac{h}{L}\ln\frac{h}{2r_w}}, \qquad (3)$$

where $$a=\frac{L}{2\sqrt{2}}\left[1+\sqrt{1+4\left(2R_{ef}/L\right)^4}\right]^{\frac{1}{2}} \qquad (4)$$

– semi-major axis of the ellipse.

S.D. Joshi [19, 20] proposed the following formula for anisotropic formation for calculating the flow rate of a horizontal well:

$$q=\frac{2\pi k_r h}{\mu B}\cdot\frac{P_{ef}-P_w}{\ln\frac{a+\sqrt{a^2-\left(L/2\right)^2}}{L/2}+\frac{\beta^2 h}{L}\ln\frac{\beta h}{2r_w}}, \qquad (5)$$

where $\beta=\sqrt{(k_r/k_b)}$ – anisotropy coefficient; k_r – horizontal formation permeability; k_b – vertical permeability.

In their works, G.I. Renard and J.M. Dupuy [19, 20] proposed the following formula for calculating the flow rate of a horizontal well:

$$q=\frac{2\pi kh}{\mu B}\cdot\frac{P_{ef}-P_w}{\operatorname{arch}(x)+\frac{h}{L}\ln\frac{h}{2\pi r_w}}, \qquad (6)$$

where $x=2a/L$, and the parameter a is calculated by the formula (4).

They also introduced the formation anisotropy parameter into formula (6) in order to take anisotropy into account $\beta=\sqrt{(k_r/k_b)}$

$$q = \frac{2\pi k_r h}{\mu B} \cdot \frac{P_{ef} - P_w}{\operatorname{arcch}(x) + \frac{\beta h}{L} \ln \frac{\beta h}{2\pi r_w'}}, \quad (7)$$

where $r_w' = (1+\beta)r_w/2\beta$.

It should be noted that formulas (1)–(4) and (6) can also be applied for anisotropic formations, if the following conditions are met [11–15]:

- the well length is much bigger than the formation thickness ($L >> h$),
- half of the horizontal well length is less than 90 % of the external boundary radius ($L/2 < 0{,}9R_{ef}$),
- the well length is bigger than the product of anisotropy coefficient by formation thickness ($L > \beta h$).

I.A. Charnyi [24] proposed the following formula when the horizontal wellbore is located symmetrically against the top and bottom of a strip formation:

$$Q = \frac{2\pi k L (P_{ef} - P_w)}{\mu B \left[\frac{2\pi H}{h} + \ln \frac{h}{2\pi r_w} \right]}, \quad (8)$$

where k – formation permeability; h – formation thickness; r_w – well radius; P_{ef}, P_w – boundary and bottomhole pressure; μ – oil viscosity; L – well length; H – distance from the well to the formation boundary.

The oil production rate of a horizontal well symmetrically located against the formation top and bottom as per [24, 25] is calculated by the following formula:

$$Q = \frac{2\pi k L \Delta P}{\mu B} \frac{1}{\left[1 + \frac{2r_w}{h - 2r_w} \ln \frac{2r_w}{h} \right] + \frac{R_{ef} - (h - 2r_w)}{2h}}, \quad (9)$$

where B – formation volume factor; $\Delta P = P_{ef} - P_w$ – differential pressure.

A.M. Pirverdyan [25], supplementing the problem statement with two conditions: one boundary $R_{ef} = R_{ef1}$ is impenetrable and the velocity vertical component is $v_y = 0$, and the second boundary $R_{ef} = R_{ef2}$ is permeable and the permeable boundary pressure is P_{ef2}, obtained the following equation:

$$Q = \frac{2\pi k L (P_{ef} - P_w)}{\mu B \left[\frac{2\pi H}{h} + \ln \frac{h}{2\pi r_w} + \frac{1}{2} \ln \frac{2}{1 - \cos \frac{\pi(2a - r_w)}{h}} \right]}, \quad (10)$$

where a – the distance from the axis of a horizontal wellbore to the top. With the symmetrical horizontal wellbore by thickness $a = h/2$.

Mostly, I.A. Charnyi [24] and A.M. Pirverdyan [25] studied fluid inflow to horizontal wells located exactly in a strip reservoir, consequently, the result of the calculation will be significantly lower than the actual one.

Determination of the Flow Rate of a Horizontal Well Located Asymmetrically against the Formation Top and Bottom

In their works, Z.S. Aliev, V.V. Bondarenko, B.E. Somov, and V.V. Sheremet [23, 26, 27] studied a horizontal well that completely penetrated the formation. The formation is homogeneous and strip-like. The location of this well against the formation top and bottom varied. In these works, the filtration zone divided into two parts was considered. In each zone near the wellbore the formation thickness was considered the radius function, i.e. $h = h(r_w)$ (Fig. 2).

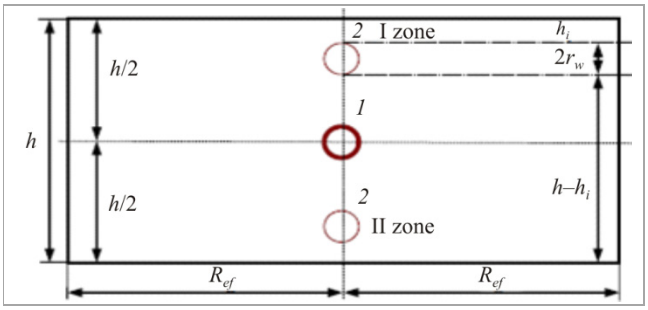

Fig. 2. Asymmetric location of the horizontal wellbore against the strip formation top and bottom

In these works, the following formula was proposed for the asymmetric well location:

$$Q = \frac{2\pi k L \Delta P}{\mu B} \cdot 1 \Big/ \frac{2}{h_1} \left[h_1 + r_w \ln \frac{2r_w}{h_1 + r_w} \right] + \frac{R_{ef} - h_1}{(h_1 + r_w)} + \frac{2}{h_2} \left[h_2 + r_w \ln \frac{2r_w}{h_2 + r_w} \right] + \frac{R_{ef} - h_2}{(h_2 + r_w)}, \quad (11)$$

where h – formation thickness; $h_i = (h - h_2) - r_w$ – formation thickness minus well radius.

In order to calculate the effect of anisotropy in formula (11) for an isotropic formation, similarly to formulas (5) and (7), it is necessary to introduce the anisotropy parameter $\beta = \sqrt{(k_r/k_b)}$;

Then the formula for the horizontal well flow rate in the anisotropic case will be as follows:

$$Q = \frac{2\pi k L \Delta P}{\mu B} \cdot 1 \Big/ \frac{2}{\beta h_1} \left[\beta h_1 + r_w \ln \frac{2r_w}{\beta h_1 + r_w} \right] + \frac{R_{ef} - \beta h_1}{(\beta h_1 + r_w)} + \frac{2}{\beta h_2} \left[\beta h_2 + r_w \ln \frac{2r_w}{\beta h_2 + r_w} \right] + \frac{R_{ef} - \beta h_2}{(\beta h_2 + r_w)}. \quad (12)$$

The symmetrical location of a horizontal well against the formation top and bottom with different pressures on the external boundary is considered in the works of V.P. Pilatovsky [7, 8]. If $P_{ef1} = P_{ef2} = P_{ef}$ and the wellbore is symmetrical against the external boundaries, then:

$$Q = \frac{2\pi k L \Delta P}{\mu \left[\frac{R_{ef}}{2h} - \frac{1}{2\pi} \ln \left(\frac{2\pi r_w}{h} \sin \frac{\pi a}{h} \right) \right]}, \quad (13)$$

where $a = h/2$.

There is also a completely different approach for determination of the horizontal well productivity. It consists in dividing the filtration area into external and internal zones. In this case, the flow simulation depends on the filtration zones: the external zone is described by a flat two-dimensional flow, where the filtration conditionally occurs in the horizontal plane; the internal zone is characterized as a three-dimensional flow, the drainage zone is an ellipsoid of revolution simulating a horizontal wellbore with a radius r_w. This approach was described in detail by V.P. Merkulov [28–30] and has the following form:

$$Q=\frac{2\pi khL\Delta P}{\mu\left\{\left[\frac{\pi b}{h}+\ln\frac{h}{2\pi r_w}-\left(\ln\frac{a+c}{2c}+\lambda\right)\right]+\frac{L}{h}\ln\frac{2R_{ef}}{(a+b)}\right\}}. \quad (14)$$

According to V.P. Merkulov [28–30], for the wellbore asymmetric location against the drainage zone center at a distance δ the dependence (14) will be as follows:

$$Q=2\pi khL\Delta P\Big/\mu\left\{\left[\frac{\pi b}{h}+\ln\frac{h}{2\pi r_w}-\left(\ln\frac{a+c}{2c}+\lambda\right)\right]+\right.$$
$$\left.+\frac{L}{h}\ln\frac{2R_{ef}}{(a+b)}\left(1-\frac{\delta^2}{R_{ef}^2}\right)\right\}, \quad (15)$$

where L – horizontal wellbore length; $a = 0.5L + 2h$ – semi-major axis of the ellipse; $b = (2Lh + 4h^2)^{1/2}$ – semi-minor axis of the ellipse; $c = 0.5L$ – focus distance; r_w – well radius characterizing the flow zone of radial wellbore in relation to the horizontal wellbore; $\lambda = f(\alpha, \omega)$ where $\alpha = L/2h$, $\omega = \delta/h$, δ – eccentricity of the asymmetric location of the horizontal wellbore axis by thickness.

Under the conditions of a circular anisotropic formation V.P. Merkulov [29] proposed to calculate the horizontal well flow rate by the formula

$$Q=2\pi khL^*\Delta P\Big/\left\{\mu\beta\left\{\left[\frac{\pi b^*}{\beta h}+\ln\frac{\beta h}{2\pi r_w}+\right.\right.\right.$$
$$\left.\left.\left.+\left(\ln\frac{a^*+b^*}{2c^*}+\lambda^*\right)\right]+\frac{L^*}{\beta h}\ln\frac{2R_{ef}}{(a^*+b^*)}\right\}\right\}, \quad (16)$$

where $a^* = 0.5L + 2\beta h$; $b^* = (2L\beta h + 4h^2)^{0.5}$; $c^* = 0,5L$; $\lambda^* = f(a^*)$, where $a^* = L^*/2h$, under these conditions $L^* = L$.

In his works, S.D. Joshi [19, 20] also divided the flow into zones, but before determining the dependence of the horizontal well flow rate Q on the drawdown ΔP, he proposed to find the sum of filtration resistances arising during the fluid flow in these planes. The displacement of the horizontal wellbore against the formation top and bottom takes into account the parameter λ_δ characterizing the distance from the wellbore to the conditional center:

$$Q=\frac{2\pi kh\Delta P}{\mu B\left[\ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right)+\frac{h}{L}\ln\frac{(h/2)^2-\lambda_\delta^2}{hr_w/2}\right]}. \quad (17)$$

Taking into account anisotropy:

$$Q=\frac{2\pi kh\Delta P}{\mu B\left[\ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right)+\frac{\beta^2 h}{L}\ln\frac{h}{2r_w}\right]}. \quad (18)$$

In their works, G.I. Renard and J.M. Dupuy [21, 22] proposed a formula for the flow rate of a short horizontal well, i.e. if the well length L is less than the formation thickness h:

$$Q=\frac{2\pi kh\Delta P}{\mu B\left[\cosh^{-1}(X)+\frac{h}{L}\ln\frac{h}{2\pi r_w}\right]}. \quad (19)$$

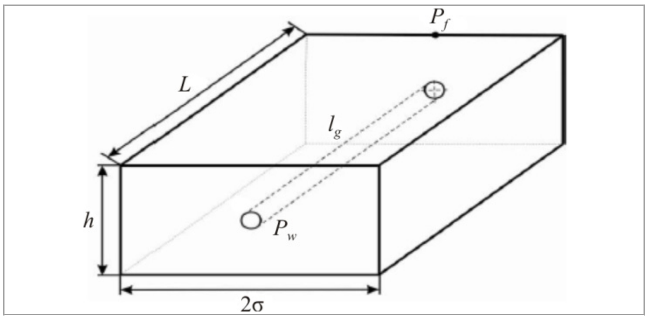

Fig. 3. Horizontal well in a rectangular oil formation

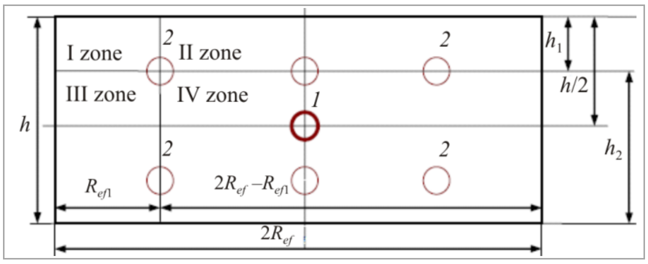

Fig. 4. Scheme of conditional location of a horizontal well in the formation: *1* – symmetric; *2* – asymmetric

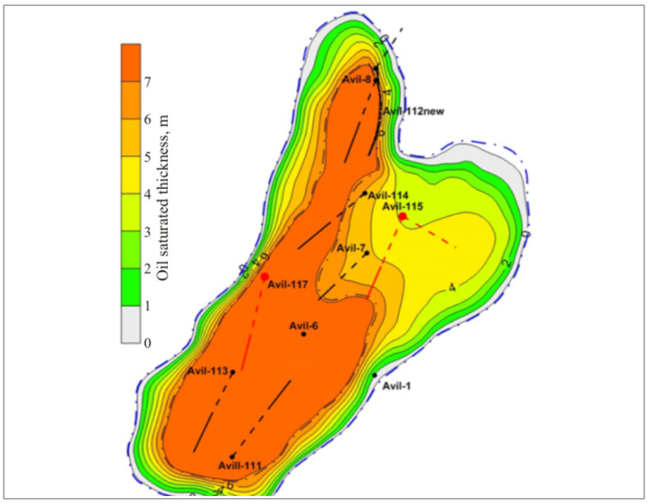

Fig. 5. Avilovskoye gas and oil field

Taking into account the anisotropy, they presented this formula as:

$$Q=\frac{2\pi kh\Delta P}{\mu B\left[\cosh^{-1}(X)+\frac{\beta h}{L}\ln\frac{\beta h}{2\pi r_w'}\right]}, \quad (20)$$

where $X = a/0,5L$, $r_w' = (1+\beta)r_w/2\beta$.

In his work, V.D. Lysenko [31] obtained an equation for determination of the horizontal well flow rate (Fig. 3), conventionally located in a rectangular oil formation with a length of $2L$, the well length equaling to the oil formation width 2σ, the formation thickness h:

$$Q=\frac{2\pi kh\Delta P}{\mu B\left(\frac{L}{4\sigma}+\frac{1}{2\pi}\ln\frac{2\sigma+l_g}{2l_g}+\frac{h}{2\pi l_g}\ln\frac{h}{2\pi r_w}\right)}. \quad (21)$$

Here, L – the distance from the well with pressure $P = P_w$ to the external boundary with pressure $P = P_{eb}$; 2σ – the formation width; l_g – horizontal wellbore length; r_w – wellbore radius.

Table 1

Initial data for calculation of the flow rate of horizontal wells for the Avilovskoye gas and oil field

Parameter	Unit of measurement	Well number						
		7	8	111	112	113	114	117
P_f	MPa	21	21	21	21	21	21	21
P_b	MPa	20	20	20	20	20	20	20
Differential pressure	Pa	1000000						
Net oil thickness h	m	2	1.5	1.4	2	0.8	1.3	1.2
h_1	m	1	1	1	1	0.4	1	1
h_2	m	1	0.5	0.4	1	0.4	0.3	0.2
R_c	m	0.073	0.073	0.073	0.073	0.073	0.073	0.073
R_k	m	298	442	371	685	398	442	521
L	m	123	257	243	293	466	287	384
Viscosity	of oil under formation conditions, Pa·s	0.000551						
	of water under formation conditions, Pa·s	0.000816						
	of gas under formation conditions, Pa·s	0.0000211						
Porosity	unit fraction	0.195						
Density under formation conditions	of oil, kg/m^3	698						
	of water, kg/m^3	1120						
	of gas, kg/m^3	0.981						
Formation volume factor	oil	1.364						
	water	1.0014						
Compressibility, 10^{-4}, 1/MPa	oil	19.2						
	water	4.35						
Gas-oil ratio	m^3/t	147.7						
Anisotropy		0.72						
H	m	500						
Permeability K, mD (10^{-3} μm ^{2})	abs	80						
	oil relative	6.983						
	gas relative	0.551						
	water relative	12.29						
	at the following concentration	Fluids in the flow: 25 % oil, 25 % water, 50 % gas						

Table 2

Results of calculation of the flow rate of horizontal wells by classical formulas

Formula number	Well No.						
	7	8	111	112	113	114	117
Fact	67.0	70.0	75.0	70.0	78.0	71.0	75.0
1	479.4	426.3	424.1	490.0	357.9	392.3	389.6
2	486.2	432.5	432.1	494.5	389.7	399.2	398.6
3	479.4	426.3	424.0	489.9	356.9	392.3	389.5
6	479.4	426.3	424.0	489.9	356.9	392.3	389.5
7	480.8	426.8	424.6	490.6	357.0	392.6	389.7
8	293.5	148.5	165.2	127.8	88.0	128.8	100.8
9	105.6	111.8	117.5	109.7	120.2	108.2	113.5
10	0.4	0.3	0.2	0.4	0.1	0.2	0.2
11	14.2	13.9	13.8	14.7	17.8	11.5	10.3
12	64.0	91.4	103.3	66.3	192.7	102.9	117.4
13	5.5	–5.6	4.0	–2.3	4.0	6.3	20.7
14	54.1	52.5	52.6	53.6	49.7	51.4	51.1
15	54.1	52.5	52.6	53.6	49.7	51.4	51.1
16	140.4	127.3	127.6	140.7	113.6	119.3	119.3
17	0.0	0.0	0.0	0.0		0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Error 1	–615.5	–509.1	–465.5	–599.9	–358.9	–452.6	–419.5
Error 2	–625.7	–517.9	–476.1	–606.4	–399.6	–462.3	–431.5
Error 3	–615.5	–509.0	–465.4	–599.9	–357.5	–452.5	–419.3
Error 6	–615.5	–509.0	–465.4	–599.9	–357.5	–452.5	–419.3
Error 7	–617.7	–509.7	–466.1	–600.8	–357.7	–453.0	–419.6
Error 8	–338.0	–112.2	–120.2	–82.6	–12.8	–81.3	–34.5
Error 9	–57.6	–59.7	–56.7	–56.7	–54.1	–52.5	–51.3
Error 1	99.5	99.6	99.7	99.5	99.8	99.7	99.7
Error 11	78.8	80.1	81.6	79.0	77.2	83.8	86.3
Error 12	4.5	–30.6	–37.8	5.3	–147.1	–44.9	–56.5
Error 13	91.8	108.1	94.6	103.3	94.8	91.1	72.4
Error 14	19.3	25.0	29.9	23.4	36.3	27.7	31.9
Error 15	19.3	25.0	29.9	23.4	36.3	27.7	31.9
Error 16	–109.5	–81.9	–70.1	–101.0	–45.6	–68.0	–59.1
Error 17	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Error 18	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Error 19	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Error 20	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Dependence of the Horizontal Well Productivity on its Asymmetric Location against the Formation External Boundaries

This problem was very well covered in the works of Z.S. Aliev, B.E. Somov, V.V. Bondarenko, B.A. Nikitin, K.S. Basniev et al. [26, 27, 32].

For appropriate conditions, the asymmetric well location was described in relation to the formation boundaries (Fig. 4). The results of the numerical solution of the equation of three-dimensional unsteady filtration showed that the well asymmetric location by the thickness of a homogeneous formation significantly reduces the horizontal well productivity. With an increase in reservoir thickness, well production losses also increase depending on the asymmetry coefficient. The maximum well productivity is achieved under the conditions of wellbore symmetrical location – against the external boundaries as well as against the formation top and bottom.

Let us try to apply these formulas (1)–(20) to the Avilovskoye field (Fig. 5).

The commercial oil and gas content of the Avilovskoye field is confined to the sandstones of the Bobrikovsky horizon (C1bb formation) of the Visean stage of the Lower Carboniferous system. One gas and oil reservoir was found.

One massive, gas-oil reservoir with low specific height, underlain by water, was revealed in the sediments. The formation is composed of sandstones. The reservoir type is porous. The physicochemical properties of oil and dissolved gas were studied on seven deep oil samples from seven wells and 16 surface samples from seven wells. The oil is of especially light density and low viscosity, paraffin, low in sulfur and tar. The physicochemical properties of gas-cap gas and condensate were studied on three samples from two wells. The gas-cap gas is methane, semi-wet, gas density by air – 0.800, hydrogen sulfide content – 1.540 % mol.

The initial data for calculation of the flow rate of horizontal wells are presented in Table 1.

Let us calculate the flow rates of horizontal wells by the classical formulas (1)–(20). The calculations results and the initial data are summarized in Table 2.

Conclusion

Based on the analysis of the information obtained, it was revealed that the following parameters affect the horizontal well productivity:

– location of the well against the formation thickness and against the drainage zone boundaries (symmetric and asymmetric);
– quality of the productive interval opening;
– wellbore profile, in particular the horizontal part;
– well design features;
– pressure losses in the horizontal part of the well;
– porosity and permeability formation properties;
– number of multistage hydraulic fracturing ports (if any), etc.

At the same time, all the formulas presented in the work were developed for a stationary fluid flow, fluid filtration occurs in an isotropic reservoir (there is a correction for vertical anisotropy), the formation top and bottom are impermeable, the fluid is incompressible, all wells have a perfectly straight horizontal wellbore. Among other things, all the above formulas are based on the Darcy's linear filtration law, there are differences in the definition of the well drainage zone geometry, and, as a consequence, this limits the area of these mathematical models application.

The large error in the calculations is due to the presence of a high gas factor; at the wells of the Avilovskoye field, the gas factor varied from 200 to 15,000 m^3/t. This leads to a natural gas lift and a transition from laminar to turbulent flow. Accordingly, it is impossible to discuss linear filtration under the conditions of this gas-oil field.

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