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## SOLVING THE PROBLEMS OF STRENGTH AND DESTRUCTION OF MATERIALS AND STRUCTURAL ELEMENTS USING A COMPLEX EXPERIMENTAL-THEORETICAL APPROACH

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### ABSTRACT

The paper considers a complex experimental and theoretical approach to studying high-speed deformation of structural materials including a system of basic dynamic experiments aimed at determining the strength and deformation properties of materials under various types of stresses, a program for direct parametric identification of models of deformation and fracture, as well as a system of special verification experiments in the natural and numerical realizations, which makes it possible to assess the adequacy of the model obtained and its performance in conditions other than those in which it was received. To obtain a set of mechanical strength and deformation properties of materials under compression, tension or shear, a series of basic experiments was carried out on an installation that implements the Kolsky method using sets of split Hopkinson bar of various configurations. According to the results of these experiments, together with the data of static deformation, the parameters of the Johnson-Cook model of plasticity with various versions of the model factor responsible for the influence of the strain rate were identified. To test the adequacy of the model (verification), special test experiments have been developed that are simple enough, on the one hand, and allow an unambiguous interpretation of the results and numerical reproduction without simplifications, whilst, on the other hand, the stress state in these types of tests, as well as the history of changes in the loading parameters, differs from that in basic experiments. The chain of obtaining an adequate (verified) model of deformation and fracture used in software complexes for calculating the stress-strain state and the strength of critical structures under shock loading conditions is considered in the example of steel 3.

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## Introduction

Elements of various constructions during operation can be subjected to intense shock or explosive loads. Numerical methods and modern computing resources allow solving with a sufficient degree of accuracy complex systems of mathematical equations describing the processes occurring in a structure under the influence of various factors, to describe the behavior of materials by complex nonlinear mathematical models, and to take into account geometric and physical nonlinearities. The adequacy of the numerical solution is mainly determined by the accuracy of the input data, which are one of the important components mathematical models of the behavior of materials. To equip and verify the phenomenological behavior models used in the calculation of the stress-strain state, an extensive database on the dynamic properties of materials is needed.

Currently, one of the most commonly used experimental methods in practice is the Kolsky method or the split Hopkinson pressure bar method (SHPB) [1]. In his work, Kolsky described a modification of the experimental Hopkinson method, the experimental procedure, and also derived equations that allow the use of measurements made during the test to determine stress and strain in the sample, as well as briefly discussed the effects of inertia and used the data to construct a modified Boltzmann model [2] for tested materials. The creation of the SHPB method has become the cornerstone in experimental science for measuring the state of a sample during dynamic loading. Since the time of the first Kolsky experiments, the researchers have proposed many modifications of the classical scheme, which allow testing samples under tension, shear, combined loading, with a change in the history of the strain rate during loading, and also cyclic and alternating loading [3–14]. In addition, to assess the adequacy of the models used, it is necessary to develop a system of special test experiments. However, as analysis of the current situation in the field of high-speed deformation of structural materials

shows, most of the work is devoted only to experimental or only theoretical research, while a successful experimental solution of the problems of high-speed deformation of materials and structures requires a comprehensive experimental-theoretical approach combining experimental studies, mathematical modeling and numerical experiment [15–21].

The paper presents a comprehensive approach to the study of high-speed deformation of materials of various physical nature, including numerical analysis of the developed techniques, obtaining dynamic properties, identification on this basis of the necessary parameters of mathematical models of materials, and verification of these models using a set of special test experiments in natural and numerical implementations.

A chain of obtaining adequate (verified) models of deformation and fracture of steel 3 used in software complexes for calculating stress-strain state and the strength of responsible structures under shock loading is considered.

## Experimental Methods and facility

To obtain a set of mechanical strength and deformation properties of materials during compression, tension and shearing, a series of basic experiments was performed at a facility that implements the Kolsky technique using sets of the split Hopkinson pressure bar of various configurations (Fig. 1). A gas gun of 20 mm caliber is used as a loading device which makes it possible to accelerate the strikers in lengths from 50 to 400 mm in the speed range 5–50 m/s, with an appropriate strain rate realized in the range  $5 \cdot 10^2 - 5 \cdot 10^3 \text{ s}^{-1}$ . For compression tests, a set of pressure bars of high-strength steel with a length of 1 m each is used. When testing low-density specimens or small-diameter samples (compared to the diameter of the bars) a set of pressure bars of different lengths is used to record several load cycles during one test [22].

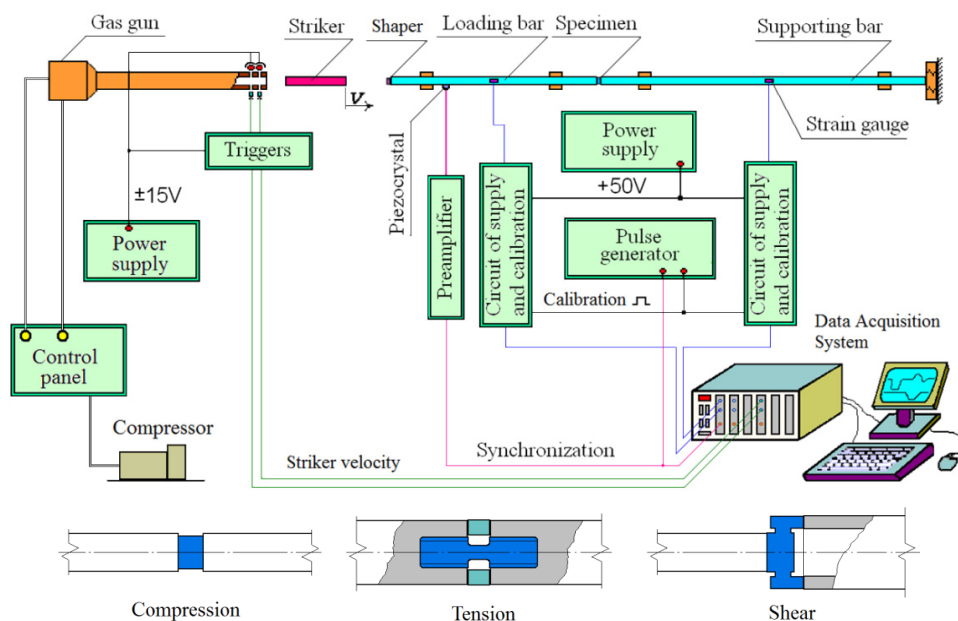


Fig. 1. The experimental setup and main test schemes

Nicholas's scheme [23] is used for carrying out basic tensile tests. As a result, a stress-strain curve is obtained with the dependence of the strain rate, as well as the ultimate strength and plasticity characteristics are determined. To obtain the properties of materials in shear, we use a specimen of special configuration (a circular plate with annular grooves), as well as a measuring tube is used instead of a support bar (Fig. 1). As a shortcoming of the test procedure for shear testing it should be noted that at high degrees of deformation the samples, in addition to shearing, also undergo tensile deformations.

The set of basic experiments listed allows to obtain mechanical properties of materials under different, but homogeneous, stressed state, at approximately constant rate of deformation and constant temperature. The results of these experiments are used for direct parametric identification of plasticity models.

For direct identification, it is assumed that the basic experiments are statically determined, i.e. it is assumed that the stress and strain fields in the working region of the test samples are homogeneous, which makes it possible to determine the stresses and strains at the points of the sample on the basis of the measured integral (force and displacement) measured in the experiment. Using this approach, the applicability of the experimental curves to identify model parameters is limited to the area of uniform deformation of the working part of the sample before the appearance of plastic deformation localization. In addition it is necessary to conduct numerous experiments to assess the influence of various factors (strain rate, temperature, type of stress-strain state, etc.) on the behavior of the material. For example, to identify a viscoplastic material model it is necessary to determine the stress-strain curves at different strain rates [24], and to determine the parameters of the fracture model it is necessary to obtain limiting characteristics for different types of stress state: values of the triaxiality index of stress state [24, 25] and Lode angles [26].

To identify the model of viscoplasticity plastic branches of deformation curves are used obtained at different strain rates and temperatures. The problem of determining the optimal set of parameters in this case is identical to the problem of approximating the experimental data in the four-dimensional space "deformation-strain rate-temperature-flow stress". In the Johnson-Cook model, the yield stress is defined as a function of deformation, strain rate, and temperature. It is necessary to define five material constants:  $A$ ,  $B$ ,  $n$ ,  $C$ ,  $m$ . As initial data, dynamic stress-strain curves obtained at different strain rates and temperatures are used. The optimization tasks were solved numerically using the *python* programming language (<https://www.python.org>) and an open library *lmfit* for solving optimization problems (<http://lmfit.github.io/lmfit-py/>).

Using the results of tensile tests fracture characteristics were determined: elongation after rupture  $\delta$ , relative narrowing after rupture  $\psi$  and temporal tensile strength  $\sigma_B$ . These data were used to equip the fracture model.

In order to fully verify the adequacy of model parameters determined from experiments it is necessary to verify them but in experiments of another type than those in which these parameters were obtained. For this purpose we usually use a system of a few validation experiments (natural and respective numerical ones): direct impact method (Fig. 2, *a*), modified Taylor test (Fig. 2, *b*), as well as two original modifications of the Kolsky method: high-speed indentation of indenter of various configurations (Fig. 2, *c*), and experiment on high-speed diametrical compression of cylindrical specimen of an elastoplastic material loaded in the SHPB system along the generatrix (Fig. 2, *d*). Verification experiments on the one hand are simple enough and allow unambiguous interpretation of the results and numerical reproduction without simplifications, and on the other hand – the stress state in these types of tests, as well as the history of the loading parameters change differs from that in the basic experiments. All numerical experiments (except for diametrical compression) were modeled in an axisymmetric statement and their schemes corresponded to similar physical tests. In this work, to verify the steel model, an experiment on diametrical compression of a sample was used.

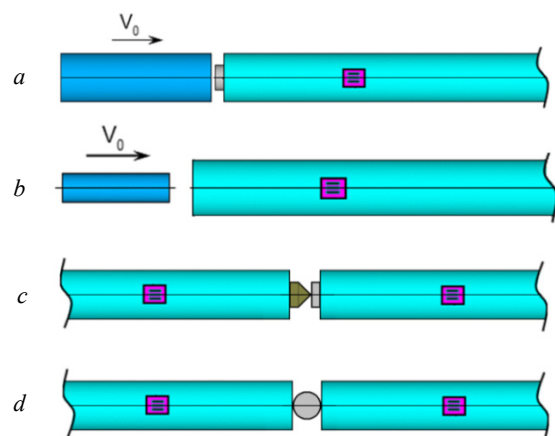


Fig. 2. Experimental schemes for validation of material model

The advantage of the proposed and used set of verification experiments is that, in addition to determining the residual irreversible shaping of the specimens (depth and diameter of the imprint, change in length, diameter, etc.), deformation processes are obtaining from the measuring bars. The data determined in the course of actual experimental studies are compared with the results of numerical simulation of the corresponding experimental schemes. The adequacy of the material model is assessed by the degree of conformity of the results.

## Investigated materials and Results

The described chain of obtaining a verified model of deformation of a material is considered in the example of the study of steel 3. For compression tests specimens were made in the form of cylinders with a diameter of 7 mm and a diameter of 14 mm. For tensile tests the specimens with

threaded heads had a working part length of 10 mm and a diameter of 5 mm.

In the complex of studies a series of basic dynamic experiments were performed for compression and tension at different strain rates and at temperatures of +20 °C, +150 °C and +350 °C. The change in the strain rate of the specimen was ensured by varying the speed of the impactor, and the required test temperature was provided by heating the ends of the measuring bars and the specimen placed between them by using a special furnace. As a result the stress-strain curves and the dependence of the change in strain rate are obtained. At each mode 3-5 tests were carried out, the results of which were then averaged.

Fig. 3 shows the averaged dynamic stress-strain curves (solid lines) and the corresponding strain rate change curves (dashed lines in the lower part of the figures) obtained under different conditions for strain rate and temperature.

To determine the limiting characteristics of plasticity (relative elongation and relative narrowing after rupture) the connected halves of the broken specimen were measured with a microscope and a digital eyepiece-camera. The obtained limiting characteristics of ductility are shown in Fig. 4.

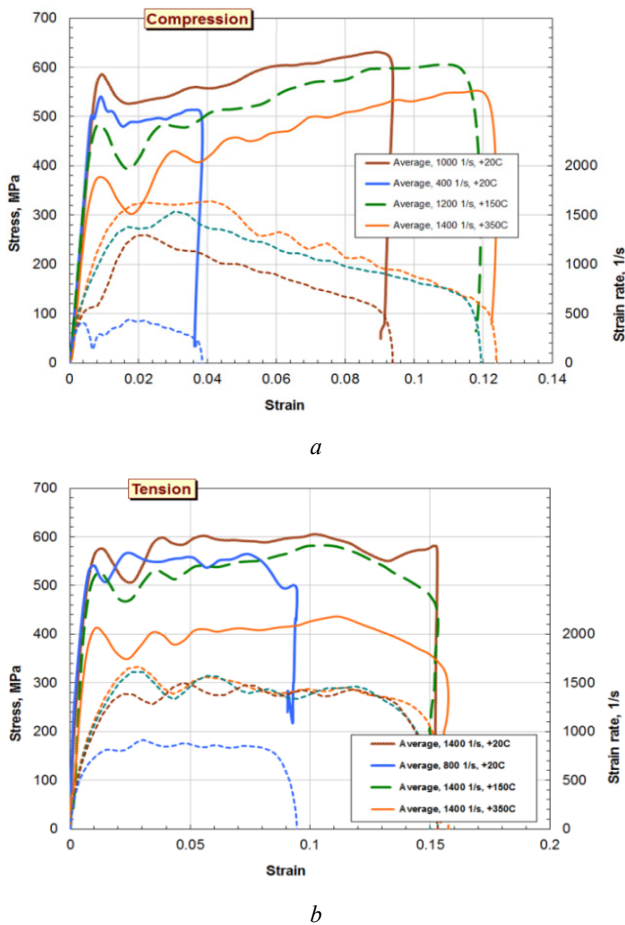


Fig. 3. The effect of strain rate and test temperature on the stress-strain curves under compression (a) and tension (b)

Numerical modeling of the processes of dynamic deformation of the samples in order to verify the constitutive relations was carried out using the well-known software

package LS-DYNA. An explicit time integration scheme was used in the calculations [27]. In this case, the time step is chosen by the solver based on the stability condition of the explicit numerical solution in accordance with the Courant-Friedrichs-Lewy criterion [28] and is determined by the geometric dimensions of the finite elements and the physical characteristics of the materials [27]. To discretize the space, we used finite elements (axisymmetric or three-dimensional, depending on the simulated experiment) with one integration point in Lagrange variables (SECTION\_SHELL with parameter ELFORM=14 for axisymmetric elements and SECTION\_SOLID with parameter ELFORM=1 for three-dimensional finite elements [29]).

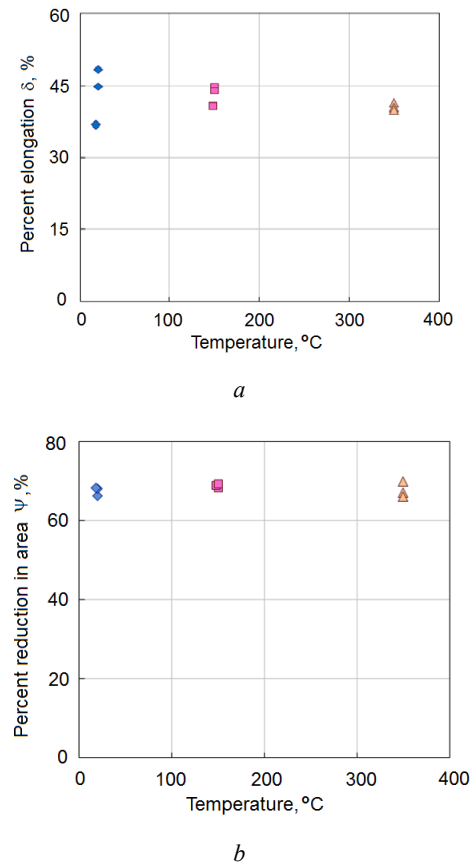


Fig. 4. Limit values of plasticity of steel 3: relative elongation (a) and relative narrowing (b)

Based on the results of realized dynamic experiments, together with the static tests data, the parameters of the Johnson-Cook plasticity model [27] were identified in which the yield stress is defined as a function of deformation, strain rate and temperature, and has the following form:

$$\sigma_{JC} = (A + B\varepsilon_p^n) (1 + C \cdot \ln \dot{\varepsilon}^*) (1 - T^{*m}).$$

The expression in the second brackets describes the effect of strain rate. Since for many materials the slope of the stress-strain curve, due to the adiabaticity of the deformation process, decreases with increasing strain rate the use of the standard approach in which the strain hardening parameters  $A$ ,  $B$  and  $n$  are determined from the curve obtained at



$\dot{\varepsilon} = 1 \text{ s}^{-1}$  leads to the fact that the model cannot adequately describe the experimental data in the dynamic range of strain rates. In the latest LS-DYNA releases, it became possible to use alternative forms of recording a strain-rate multiplier. Several variants of the model multiplier are known which are responsible for the effect of the strain rate. In addition to the classical (log-linear strain rate) multiplier, other variants of the strain-rate multiplier can be used:

- $1 + \left(\frac{\dot{\varepsilon}^*}{C}\right)^{\frac{1}{p}}$  – from Cowper-Symonds model [28];
- $1 + C \cdot \ln(\dot{\varepsilon}^*) + C_2 \cdot \ln(\dot{\varepsilon}^*)^2$  – from Huh and Kang model [29];
- $(\dot{\varepsilon}^*)^C$  – from Allen, Rule and Jones model [30].

To identify the Johnson-Cook model with strain-rate multipliers in the forms [27–30] (hereinafter referred to model 1-model 4, respectively) the data of the conducted experimental studies were used. Parameters of the model of steel 3 obtained during the solution of the optimization problem are summarized in Table. A model that gives the best approximation for the given material (minimum deviation of the mathematical curve from the experimental deformation diagrams) is indicated by color.

Model parameters for steel 3

Parameter	Model 1	Model 2	Model 3	Model 4	Dimension
A	412.5	192	394	412	MPa
B	1201	570	1136	1223	MPa
n	0.83	0.83	0.83	0.83	–
C	2.18E-02	3.61E-02	1.74E-02	2.0E-02	–
C <sub>2</sub>	–	–	0.00184	–	–
p	–	26.758	–	–	–
m	0.999	0.993	0.983	0.996	–

Since the data of "basic" experiments (homogeneous and uniaxial stress state, constant strain rate and temperature) are used to identify mathematical models, it is required to check the operability of the equations of state in real conditions of operation of structural units. To test the adequacy of the model, special verification experiments have been developed that are, on the one hand, rather simple and allow unambiguous interpretation of the results and numerical reproduction without simplifications, and on the other hand, the stress state in these types of tests and also the history of loading parameters change differs from that in basic experiments. Possible variants for such verification experiments were shown in Fig. 2.

Below are the results of verification of model 1 for steel 3 using the experiment on dynamic diametrical elastoplastic compression of the specimen (Fig. 2, d) in natural (physical) and numerical implementations. Samples in the form of tablets 14 mm in diameter and 7 mm in length were subjected to deformation. The behavior of the sample material was described by a verifiable mathematical model,

the parameters of which were determined from the results of basic experiments on simple tension and compression.

The SHPB system was loaded, as in the case of the full-scale test, by the impact of a short (300 mm long) striker 20 mm in diameter and made of the same material as the measuring bars. The initial velocity of the striker was set as an initial condition (INITIAL\_VELOCITY\_GENERATION control card [29]) and corresponded to the velocity in the full-scale test.

In full-scale tests, all surfaces of the contact "measuring bar-specimen-indenter" were lubricated to reduce friction, and the ends of the bars and samples were polished. Since the exact value of the coefficient of friction on the contact surfaces (especially in the case of dynamic loading) is unknown and the effect of friction was minimized as much as possible, in virtual experiments frictionless contact was used between the parts of the model. All external surfaces of individual parts of the model were considered contact (CONTACT\_AUTOMATIC\_SINGLE\_SURFACE [29]).

To register deformations in a certain section of the measuring bars, design sensors (control charts DATABASE\_ELOUT and DATABASE\_HISTORY\_SOLID (SHELL) [29]) were installed by analogy with strain gages in a full-scale test.

The grid convergence was not investigated separately; however, the small "calculated size" of the problems being solved made it possible to initially select the size of the finite element, which allows obtaining a sufficiently accurate solution. This was confirmed by solving problems on meshes with two sizes of finite elements differing by two times. At the same time, the simulation results practically coincided.

The difference between this verification experiment and those stated above, among other things, is that in this case the modeling problem is not axisymmetric and must be solved in a three-dimensional statement. Comparison the residual shape of the specimens together with the strain pulses in the measuring bars allows us to estimate the degree of adequacy of the model.

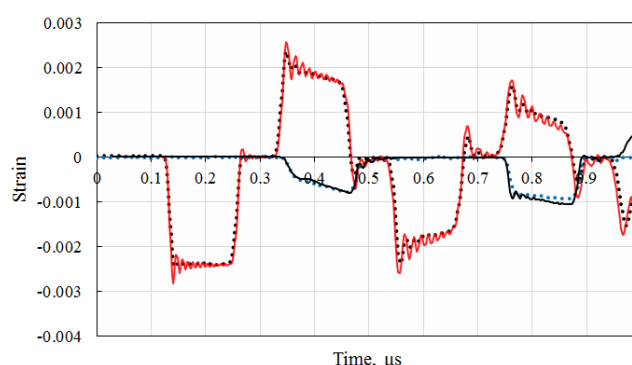


Fig. 5. Comparison of the strain pulses obtained in the calculation and registered in the experiment

As an example in Fig. 5 for steel 3 a comparison of the deformation pulses from the incident and transmitting pressure bars recorded in the experiment (markers) and obtained

in the calculation (solid line) is given. In the experiment, two load cycles are realized. Black markers and a red line represent information from the incident bar, blue markers and a black line are signals from the transmitting bar. It can be seen that the results of numerical simulation are in good agreement with the results of natural tests.

The shape of the specimen after loading obtained in the calculation (left) and in the natural experiment (right) is shown in Fig. 6. The deviation of the calculated values from the experimental data does not exceed 3%.

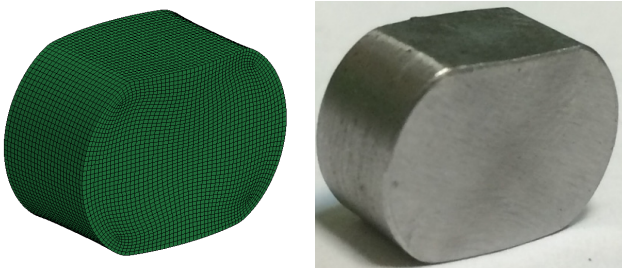


Fig. 6. Appearance of the sample after loading: left calculation, right – experiment.

In Fig. 7, *a* shows the plastic deformation fields in the specimen. As can be seen the maximum strain value is 38.5%. Fig. 7, *b* illustrates the temperature deviation at the sample points during the adiabatic deformation heating from the initial value. The maximum temperature in the center of specimen increased up to 85 °C. Such an increase in temperature cannot lead to significant changes in the mechanical characteristics of the steel; however, at higher degrees of plastic deformation due to the adiabaticity of the deformation process, the increase in the sample temperature can reach hundreds of degrees [34]. Therefore, taking into account the temperature effects in the problems of intense shock deformation of structural elements is important.

Thus the complex experimental-theoretical approach used allows obtaining an adequate (verified) model of deformation of structural materials that can be successfully

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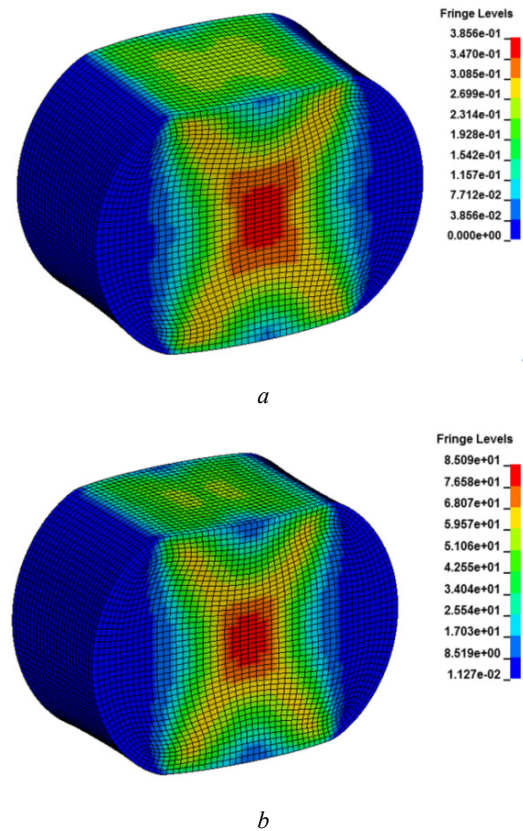


Fig. 7. The fields of plastic deformations (*a*) and adiabatic heating of the specimen during plastic deformation (*b*)

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