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OPTIMIZATION OF LAYERED SHELL STRENGTH AND GEOMETRICAL PARAMETERS, ASSESSING MECHANICAL PROPERTIES OF THE MATERIALS

ОПТИМИЗАЦИЯ ГЕОМЕТРИЧЕСКИХ ПАРАМЕТРОВ И ПРОЧНОСТИ СЛОИСТОЙ ОБОЛОЧКИ С УЧЕТОМ МЕХАНИЧЕСКИХ СВОЙСТВ МАТЕРИАЛОВ

In the design stage of layered construction it is important to find the optimal geometrical parameters and to choose materials. This article used the Lagrange method to calculate the reduced stress limit under a complex stress state and estimated the optimal thickness of construction. The calculations were performed for glass plastic, carbon plastic and layered shell constructions. An efficient selection of geometrical parameters is presented in this article.

Keywords: cylindrical shell, limit stress, optimal thickness, fiberglass composite, carbon fiber composite, layered metal construction.

В стадии проектирования слоистой конструкции важно найти оптимальные геометрические параметры и выбрать материалы. Использован метод Лагранжа для расчета редукционного предельного напряжения при сложном напряженном состоянии и оптимальной толщине конструкции. Расчеты проведены для стеклопластика, углепластика, оболочных слоистых конструкций. Представлен эффективный подбор геометрических параметров.

Ключевые слова: цилиндрическая оболочка, предельное напряжение, стекловолоконный композит, углепластик, слоистые металлические конструкции.

Introduction. Constructions made of laminated materials are lighter than those made of alloys or steel. In addition, there is a greater possibility to optimize the construction and make rational choices of the layered material. During the optimization, the layer structure of the layered materials must be taken into account during an analysis and assessment of the strength criteria that define the behavior of layered materials considering anisotropy and combined load. Because of that it is important to evaluate the mechanical properties of the materials in the cases uniaxial and biaxial tensile stress test in order to determine the optimization parameters.

Recently numeric methods have been widely used in solving optimization tasks [1, 2]. These methods help to get the results quickly and also allow evaluation of parameters that are not possible to asses using other methods. However evaluation of mechanical properties rests on oversimplified strength and deformation criteria [2, 3]. This is related to the fact that different laboratories have different equipment used to determine the mechanical properties of materials [4–6]. The von Mizes criteria used do not represent the mechanical properties of the materials rather showing the properties of a complex tension state. The best assessment of mechanical characteristics of materials is represented by Tsai – Wu strength criteria [7] but it requires measuring up to six mechanical characteristics. It is therefore appropriate to look for the strength criteria that is both accurate and requires fewer measurements.

In this research we use classical Lagrange optimization method [8–10], and for evaluating the strength we use our proposed strength criteria designed for composite materials. The validity of criteria is backed up by experimental research [11]. The advantage of the criteria is that it requires less experimental research and they are done easier in laboratories.

1. *Optimization of geometric dimensions for a pressure loaded cylindrical element in assessing a complex stress state.* The forces and stresses on a cylinder, which is under an internal pressure of p can be written as

$$N_x = pR, \quad N_y = \frac{1}{2} pR; \quad \sigma_1 \neq 0; \quad \sigma_2 \neq 0$$

and $\sigma_y = \frac{1}{2}\sigma_x$; N_x, N_y – axial forces; σ_x, σ_y – normal stress in direction of axis x and y.

Stresses σ_x and σ_y are the principal stresses, and σ_1, σ_2, acting at any angle ϕ_i refers to fiber directions 1 and 2 – τ_{12} shear stresses (fig. 1).

In this way, forces for the unit element width will be calculated [9]:

$$N_x = \sum_{i=1}^{k} \sigma_x^{(i)} h_i = \sum_{i=1}^{k} \sigma_1^{(i)} h_i \cos^2 \phi_i + \sum_{i=1}^{k} \sigma_2^{(i)} h_i \sin^2 \phi_i - \sum_{i=1}^{k} 2\tau_{12}^{(i)} \sin\phi_i \cos\phi_i h_i,$$

$$N_y = \sum_{i=1}^{k} \sigma_y^{(i)} h_i = \sum_{i=1}^{k} \sigma_1^{(i)} h_i \sin^2 \phi_i + \sum_{i=1}^{k} \sigma_2^{(i)} h_i \cos^2 \phi_i + \sum_{i=1}^{k} 2\tau_{12}^{(i)} \cos\phi_i \sin\phi_i, \quad (1)$$

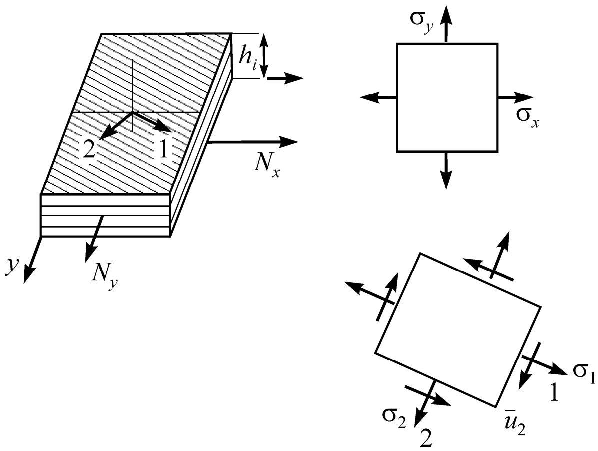

Fig. 1. Stress state of the layered element

here

$$\sigma_1^{(i)} = E\varepsilon_1^{(i)} = E\left(\varepsilon_x \cos^2\phi_i + \varepsilon_y \sin^2\phi_i\right),$$

$$\sigma_2^{(i)} = E\varepsilon_2^{(i)} = E\left(\varepsilon_x \sin^2\phi_i + \varepsilon_y \cos^2\phi_i\right),$$

$$\tau_{12} = G\gamma_{12} = 2G\left(\varepsilon_x - \varepsilon_y\right)\sin\phi_i \cos\phi_i .$$

$$E = \frac{1}{2\left(E_1 + E_2\right)}\left[2E_1E_2 + \frac{E_1^2\left(1-\nu_{12}^2\right) + E_2^2\left(1-\nu_{21}^2\right)}{1-\nu_{12}\nu_{21}}\right],$$

$$\nu = \frac{E_1\nu_{12} + E_2\nu_{21}}{E_1 + E_2},$$

$$G = \frac{E}{2\left(1+\nu\right)}.$$

In the calculation of the allowable height, we note that it is composed of several layers, i.e.

$$h = \sum_{i=1}^{k} h_i .$$

Applying the Lagrange method and the multiplier λ, we will find the minimization function, L. Before that, the complex stress state is turned into a reduced stress state, i.e. in place of σ_x, σ_y, σ_{red} will act.

The reduced σ_{red} is calculated based on the strength theory [11]:

$$m_1\sigma_i + m_2\sigma_0 = \sigma_{\text{red}},$$

here σ_i – stress intensity factor; σ_0 – average stress.

Stress intensity

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + \sigma_x^2 + \sigma_y^2} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2} \tag{2}$$

$$\sigma_0 = \frac{\sigma_x + \sigma_y}{3}, \tag{3}$$

$$m_1 = \frac{\sigma_{U_{t,12}} + \sigma_{U,t1}}{2\tau_U}, \tag{4}$$

$$m_2 = \frac{2\tau_U - \left(\sigma_{Ut,12} + \sigma_{U,t_1}\right)}{\tau_U}. \tag{5}$$

Then, σ_i, σ_0 and m_1, m_2 were determined from the four tests: tension $\sigma_{U,t1}$, $\sigma_{Ut,2}$ double force tension (when $\sigma_x = 2\sigma_y$) $\sigma_{Ut,12}^2$ and torsion τ_U, we get σ_{red} [11]:

$$\sigma_{\text{red}} = E\varepsilon_{\text{red}}^{(i)} = E\left(\varepsilon_x \cos^2\phi_i + \varepsilon_y \sin^2\phi_i\right). \tag{6}$$

The further minimization procedure will be the following [10]:

$$L = \sum_{i=1}^{k} h_i + \lambda_x\left(N_x - \sum_{i=1}^{k}\sigma_{1,red}^{(i)} h_i \cos^2\phi_i\right) +$$
$$+\lambda_y\left(N_Y - \sum_{i=1}^{k}\sigma_{\text{red}}^{(i)} h_i \sin^2\phi_i\right) + \sum_{i=1}^{k}\lambda_i\left[\sigma_i - E\left(\varepsilon_x\cos^2\phi_i + \varepsilon_y \sin^2\phi_i\right)\right].$$

Then

$$\frac{\partial L}{\partial h_i} = 0, \quad \frac{\partial L}{\partial \phi_i} = 0, \tag{7}$$

$$\frac{\partial L}{\partial \lambda_{\text{x}}} = \frac{\partial L}{\partial \lambda_y} = \frac{\partial L}{\partial \lambda_i} = 0.$$

Differentiating the formula (7) we get

$$\sigma_{\text{red}}^{(i)}\left(\lambda_x \cos^2\phi_i + \lambda_y \sin^2\phi_i\right) = 1, \tag{8}$$

$$h_i\sigma_{\text{red}}^{(i)}\left[\left(\lambda_y - \lambda_x\right)\sin 2\phi_i\right] = E\lambda_i\left[\varepsilon_y - \varepsilon_x\right]\sin 2\phi_i.$$

Solving the equation (8) we get

$$\lambda_x = E\varepsilon_x \frac{\lambda_i}{h_i\sigma_{\text{red}}^{(i)}}, \qquad \lambda_y = E\varepsilon_y \frac{\lambda_i}{h_i\sigma_{\text{red}}^{(i)}}.$$

From that follows

$$\frac{\lambda_i}{h_i\sigma_{\text{red}}^{(i)}} = \frac{\lambda_x}{E\varepsilon_x} = \frac{\lambda_y}{E\varepsilon_y} = \frac{1}{c^2}, \tag{9}$$

here c is constant.

According to equations (8) and (6)

$$\left(\sigma_{\text{red}}^{(i)}\right)^2 = c^2.$$

We get $\sigma_{\text{red}}^{(i)} = \pm c.$

Summing the first two (1) equations and inserting the constant c we get

$$h = \frac{1}{\sigma_{\text{red},U}}\left(N_x + N_y\right).$$

So, we get the minimum value h when c is equal to the tensile strength $\sigma_{\text{red},U}$:

$$h = \frac{1}{\sigma_{\text{red},U}}\left(N_x + N_y\right). \tag{10}$$

In assessing formula (1) and eliminating $\sigma_{\text{red},U}$ from formula (10), we get

$$\sum_{i=1}^{k} h_i\left(N_x \sin^2\phi_i - N_y \cos^2\phi_i\right) = 0, \tag{11}$$

$$\sum_{i=1}^{k} h_i\left[\left(N_x + N_y\right)\sin\phi_i \cos\phi_i\right] = 0. \tag{12}$$

So there, $2k$ must also comply with the h_i and ϕ_i values of the three equations (10)–(12).

All potential optimal layered structures have the same overall thickness h, equation (9).

The optimal layered structure will be when we get the same stresses and strains in all of the layers.

The angle does not change under load.

As a result, when we introduce the new variables

$$\bar{h}_i = \frac{h_i}{h}, \quad n_y = \frac{N_y}{N_x}, \quad \lambda = \frac{1}{1+n_y},$$

then $\sum_{i=1}^{k} \bar{h}_i = 1.$

According to equations (10)–(12) and the structural parameters, we can write the optimal thickness of the layered structure in the following form

$$h = \frac{N_x}{\lambda \sigma_{\text{red}}},$$

$$\sum_{i=1}^{k} \bar{h}_i \cos^2 \phi_i = \lambda,$$

$$\sum_{i=1}^{k} \bar{h}_i \sin^2 \phi_i = \lambda n_y.$$

Under biaxial tension

$$N_x = N_y = N, \quad n_y = 1, \quad \lambda = 0{,}5$$

and

$$h = \frac{2N}{\sigma_{\text{red},u}}, \qquad \sum_{i=1}^{k} = \bar{h}_i \cos 2\phi_i = 0. \tag{13}$$

In a cylinder, when $N_x = \frac{1}{2} pR, \quad N_y = pR, \quad h_y = 2, \quad \lambda = \frac{1}{3}$

$$h = \frac{3pR}{2\sigma_{\text{red},u}}. \tag{14}$$

The formula (13) shows that in the case of a continuous material, when $h = \frac{pR}{\sigma_{\text{red}}}$, the layered cylinder thickness is 1,5 times higher than continuous.

However, the layered structure is much lighter than cylinders made of continuous materials.

2. ***Experiments.*** A glass plastic and carbon plastic experimental investigation was carried out first. It used tension machine 1253Y-2 and the test specimens shown in fig. 2.

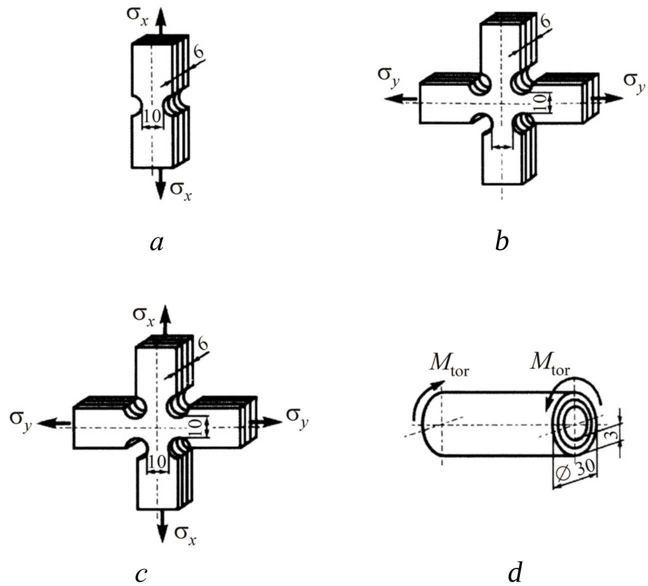

Fig. 2. Specimens and directions of stress: $a - \sigma_x \rightarrow \sigma_{Ut,1}$, $b - \sigma_y \rightarrow \sigma_{Ut,2}$, c – biaxial tension $\sigma_x = 2\sigma_y \rightarrow \sigma_{Ut,12}$, $d - M_{\text{tor}} \rightarrow \tau_U$

After this, steel pipes were tested which were manufactured by hot and cold stamping techniques.

They were tested under the same loads as glass plastic and carbon plastic specimens.

In calculation by the formulas (2)–(5), we get values for the glass plastic of $\sigma_i = 390$ MPa, $\sigma_0 = 260$ MPa, $m_1 = 2,3$; $m_2 = 2,62$, and for the carbon plastic – $\sigma_i = 749$ MPa, $\sigma_0 = 570$ MPa, $m_1 = 1,98$, $m_2 = -1,97$.

Strength parameters are shown in table.

Strength parameters of materials

Materials	$\sigma_{Ut,1}$, MPa	$\sigma_{Ut,2}$, MPa	$\sigma_{Ut,12}$, MPa	τ_U, MPa
Glass plastic	400	380	270	145
Carbon plastic	860	850	530	350
Steel Ch18N10T (hot stamping)	820	820	380	410
Steel Ch18N10T (cold stamping, with longitudinal welding)	560	610	320	330

Then we get values for the glass plastic of $\sigma_{\text{red},u} = 216$ MPa; carbon plastic – $\sigma_{\text{red},u} = 360$ MPa.

Steel Ch18N10T (pipe without welding) – $\sigma_i = 820$ MPa, $\sigma_0 = 546$ MPa, $m_1 = 2$, $m_2 = -2$, $\sigma_{\text{red},u} = 548$ MPa.

Steel (pipe with longitudinal welding) – $\sigma_i = 586$ MPa, $\sigma_0 = 390$ MPa, $m_1 = 1{,}33$, $m_2 = -0{,}66$, $\sigma_{\text{red},u} = 522$ MPa.

We compare the calculated thickness for these materials. According to the formula (14) for glass plastic

$$h_s = \frac{3pR}{2\sigma_{\text{red}}} = 6{,}9 \cdot 10^{-3} p_{\lim} R,$$

and for carbon plastic

$$h_a = \frac{3pR}{2\sigma_{\text{red}}} = 4{,}2 \cdot 10^{-3} p_{\lim} R,$$

there $p_{\lim}$ – pressure limit.

We get $h_s / h_a = 1{,}6$ i.e., the composite of carbon plastic is 1,6 times thinner than that made from glass plastic at the same pressure, steel Ch18N10T (hot stamping without welding) $h_p = 27 \cdot 10^{-3} p_{\lim} R$, $h_{ps} = 29 \cdot 10^{-3} p_{\lim} R$ (cold stamping with welding).

Thus, the steel cylinder is thinner than the glass plastic by 4 times and for carbon plastic 2,47 times. The welded pipe is 1,074 thicker than the smooth pipe.

It is known that the glass plastic density is 2,4 g/m^3, carbon plastic – 3,5 g/m^3, and steel – 7,8 g/m^3. Then, with the same pressure limit and diameter, the unit of cylinder mass

$$m_s = 16{,}56 \cdot 10^{-3} p_{\lim} R,$$
$$m_a = 14{,}7 \cdot 10^{-3} p_{\lim} R,$$
$$m_p = 21{,}26 \cdot 10^{-3} p_{\lim} R,$$
$$m_{ps} = 22{,}8 \cdot 10^{-3} p_{\lim} R.$$

So, the lightest structure can be obtained by producing it from carbon plastic material.

Conclusions. In using the Lagrange method for optimal design, the thickness minimization function is expressed by the reduced stress in a cylindrical shell.

The minimum shell thickness was determined, when the stress does not exceed the limit values.

The stress limit was determined in the experiments using glass plastic, carbon plastic and layered steel materials for the shell constructions.

The stress limit was determined by axial stress applied in different directions and in the cases of double tension and turning.

The experiments' results show that a cylindrical construction made from a composite of carbon plastic is 1,6 times thinner than that made from glass plastic, steel Ch18N10T is thinner than glass plastic by 4 times, and for carbon plastic, 2,47 times. The lightest structure can be obtained by producing it from carbon plastic material.

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